Improvement of the Integrated Weber-Fechner Law for Odors

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Abstract
The Weber-Fechner law gives the odor nuisance as the logarithm of the relation between the concentration and the odor threshold. The loss of production for an individual is the integral of the logarithm with regard to the relation. The loss can be calculated for a collective if the distribution of thresholds is known. A log-linear distribution has been used and here a log-normal distribution is tested. The log-normal gives a lower loss at high concentrations than the log-linear. The assumption that the odor in an office is both body and building odor also reduces the sensibility to odors in the model. The assumption also makes the optimal outdoor air rate less sensible to changes in the economic conditions. The theory includes percentage dissatisfied, odor thresholds, demand of improvement or odor nuisance and the loss of production. The integrated W-F law can be used to calculate the loss of production from both changes in the outdoor air rate and from new pollutants like body and carpet odor. The theory explains the increase of ventilation in Swedish schools 1879-1994.

Keywords - odor; loss; production; Weber-Fechner; economics

1. Introduction

[1] wrote in short. The perception (γ) is not related to the absolute stimulus (β). The perception is related to the logarithm of the stimulus if the stimulus is in relation to its threshold (b). In short the perception is proportional (k) to the logarithm of the relative stimulus (1).

$$\gamma = k \cdot \log \frac{\beta}{b} \tag{1}$$

The logarithm of the dose or of the concentration is used in toxicology for the dose-response relation. The loss of production as function of the concentration of indoor pollutants in an economic theory was a line in [2]. The loss of production in a collective was a curve in [3] since the individuals had a linear loss but the distribution of individual odor thresholds followed a curve. The loss of production can be given both for an individual and for the collective of workers in the space for production. The distribution of odor thresholds for the individuals in the collective must be known. A linear distribution of odor thresholds (log-linear) to the logarithm of the concentration was used in [4] since it could be integrated analytically. In this paper a normal distribution or log-normal as used in toxicology is tested. A normal distribution will give a lower threshold for the most sensible individuals than the log-linear distribution Fig. 1. The distribution of dissatisfied is used as the distribution of odor thresholds.

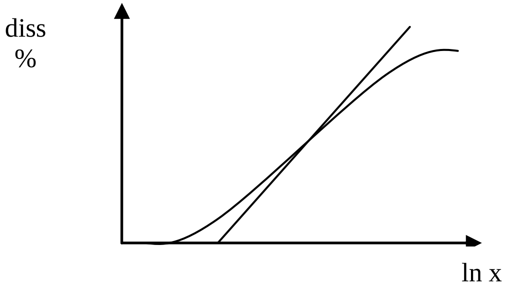

Fig. 1 Distribution of dissatisfied, linear and normally distributed to the logarithm of x

2. Loss for an Individual

The loss of production in hours Lossh(x) for an individual from an odor is described with the factor a and the relative concentration x. x is the quota of the concentration c and the odor threshold c* ppm. x is also the number of dilutions to the threshold. The outdoor air rate that dilutes the odor from a source to the threshold is q* l/s source. The outdoor air rate is q l/s source (2). The loss of production for an individual is the integral of Weber-Fechners law with regard to x (3).

a in (3) is for an individual. For a collective the average a h/yr pers is used. The q* or c* follows the distribution of thresholds and can be found with questions about dissatisfied or about odor thresholds.

$$x = \frac{c}{c*} = \frac{q*}{q} \tag{2}$$

$$Lossh\,(x) = a \cdot \int_1^x \ln x \cdot dx = a \cdot \left(x \cdot \ln x - (x-1)\right) \tag{3}$$

3. Distribution of Odor Thresholds

The concentration c_1 of substance 1 with the source strength f_1 is (4)

$$c_1 = \frac{f_1}{q} \qquad (4)$$

The odor thresholds are used to describe the odor. The number of dilutions x_{1i} to the threshold for odor 1 for the i % sensible individual is (5). At the odor threshold for person i and substance 1 $x_{1i} = 1$.

$$x_{1i} = \frac{f_1}{c^*_{1i} \cdot q} = \frac{q^*_{1i}}{q} \qquad (5)$$

The number of dilutions x_{2i} to the threshold for substance 2 and for the i % sensible is (6). At the odor threshold for person i and substance 2, $x_{2i} = 1$. The air rate at the odor threshold q^*_{2i} i % is different than for substance 1 because the relation between source strength f_2 and odor threshold c^*_{2i} for substance 2 is different than for odor 1.

$$x_{2i} = \frac{f_2}{c^*_{2i} \cdot q} = \frac{q^*_{2i}}{q} \qquad (6)$$

For body odor the thresholds is assumed to follow (7) if $q > 0.32$ l/s standard person. (7) has been found from questions about satisfied or dissatisfied [5].

$$PD = 395 \cdot e^{-1{,}83 \cdot q^{0{,}25}} \qquad (7)$$

The distribution of dissatiesfied PD at first entrance in (7) is a normal distribution to the natural logaritm of the relative concentration ln(x) or log-normal. The odor threshold for body odor for the 1,3 % most sensible person corresponds to 50 ppm CO_2 above outdoor if f = 16 l/h pers CO_2. The dissatiesfied follows the normal distribution N(ln x, 4, 1,8). There are 50 % dissatiesfied at ln x = 4 (x = e^4 = 54,6) and the standard deviation for ln x round ln x = 4 is 1,8.

The distribution of dissatisfied for other odors can be found with the same methods. The distribution is assumed to be a log-normal function N(ln x, average, standard deviation). In Table 1 there are the dissatisfied in three odors. Body and body+carpet odor has been used in work rate studies or in measurements of loss of production [6].

Table 1. Percentage dissatisfied in three odors

Outdoor	Dissatisfied in		
air rate	body	carpet	b+c
	norm (7)	[6]	[6]
l/s pers	%	%	%
3	36	38	58
10	14	32	29
30	4	15	29

In body odor from 1 pers 58 % is dissatisfied at 1,1 l/s pers (7). $q^*_{b,58\%}$ is the outdoor air rate that makes 58 % dissatisfied in body odor or the body odor threshold in flow units for the 58 % person (8).

$$1 = \frac{q^*_{b,58}}{1{,}1} \qquad (8)$$

The outdoor air rate necessary to dilute b+c odor to the threshold for the 58 % person is 3,0 l/s pers Table 1 and (9).

$$1 = \frac{q^*_{b+c,58}}{3{,}0} \qquad (9)$$

b+c odor needs 2,7 times more air to reach the odor threshold for the 58% person than for b odor (10).

$$\frac{q^*_{b+c,58}}{q^*_{b,58}} = \frac{3{,}0}{1{,}1} = 2{,}7 \qquad (10)$$

29 % diss in b+c odor at 10 l/s pers Table 1 is the same number of diss as from 1 person with b odor in 4,2 l/s (7). It is necessary with 10/4,2 = 2,4 times more air rate to dilute the odor from b+c to the threshold for the 29 % sensitive person. 29 % diss in b+c odor at 30 l/s pers is the same number of diss as from 1 person with b odor in 4,2 l/s (7). It is necessary with 30/4,2 = 7,1 more times air rate to dilute the odor from b+c to the threshold for the 29 % sensitive person. A possible reason to the difference is different odor threshold distributions in the test groups. In average b+c needs 4 times more air rate to give the same number of dissatisfied as b odor.

If the loss in a collective for other odors than mainly b odor is calculated then a for the other odor must be determined. Here a for b odor is used for b+c and body+building odor.

4. Log-linear Odor Threshold Distribution

The least sensitive in Fig. 1 line (log-linear) needs 100 times higher concentration than the most sensible to be dissatiesfied and the distribution of dissatiesfied is linear to the ln (x_d), $x_d = c_d / c^*_1$. $c^*_1 = 150$ ppm CO_2 above outdoors and the odor threshold to body odor is 0, 5 * ln x_d = ln x*.

Average loss of production for all individuals in a collective, $x_1 < x^*_n$ from [4]. The sum of demand for improvement with a log-linear distribution of diss or the loss of production in average per person as function of x_1 for the most sensible individual from x = 1 to x_1, is (11).

$$Lossh(x_1) = \frac{a}{2 \cdot \ln x^*_n} \cdot \left(x_1 \cdot \ln x_1 \cdot (\ln x_1 - 2) + 2 \cdot (x_1 - 1)\right) \quad (11)$$

Average loss of production for all individuals in a collective, $x^*_n < x_1$. Above x^*_n or above the concentration c^*_n all individuals feel the odor (12). The lossh calculated with the log-linear distribution is shown in Fig. 2.

$$Lossh(x_1) = a \cdot \left(x_1 \cdot \ln x_1 - x_1 + \frac{(x^*_n - 1)}{\ln x^*_n} - \frac{x_1 \cdot \ln x^*_n}{2} \right) \quad (12)$$

5. Log-normal Odor Threshold Distribution

The odor threshold for the most sensible is assumed to be the same concentration as where the most sensible gets dissatiesfied. This is not valid at higher numbers of dissatiesfied since the less sensible must have feelt the odor and then decided to be dissatiesfied at a higher concentration than the odor threshold. The sensitive becomes disatiesfied at the odor threshold. Here the highest percentage of dissatiesfied is 50 - 60 %.

The demand of improvement, Dem(x_1) and the loss of production Loss(x_1) is calculated as function of x_1 for the most sensible person with numericall methods and presented in Table 2 with a = 1. The diss is divided in small intervalls and the Dem and the Loss is calculated for every intervall and then the Dem and Loss is added over the intervalls. The 1,3 % most sensitive person needs 90 l/s pers not to feel body odor (7). He needs 4 times more air than 90 l/s pers not to feel body+carpet odor or 360 l/s pers. Both the demand and the loss is zero at the threshold $x_1 = 1$.

Table 2. Distribution of dissatesfied, diss at $x_1 = c/c^*_1$, $c^*_1 = 50$ ppm CO_2 above outdoor air. Demand of improvement, Dem and Loss of production, Loss at x_1 if a = 1. Oudoor air rate q l/s pers to make diss % dissatiesfied with body odor from one person.

x_1	diss	Dem	Loss	q	x_1	diss	Dem	Loss	q
	%			l/s p		%			l/s p
1	1,3	0	0	90	12	19	0,072	0,451	7,5
2	3	0,007	0,004	45	15	23	0,082	0,685	6
3	5	0,017	0,016	30	20	28	0,100	1,054	4,5
4	7	0,026	0,038	22	25	33	0,11	1,68	3,6
5	9	0,033	0,068	18	30	36	0,12	2,27	3,0
6	10	0,041	0,105	15	40	43	0,14	3,59	2,2
7	12	0,047	0,150	13	50	48	0,16	5,07	1,8
8	14	0,053	0,199	11	70	55	0,18	8,38	1,3
9	15	0,058	0,254	10	100	62	0,21	14,0	0,9
10	17	0,063	0,315	9	150	70	0,22	24,7	0,6

6. Determination of a in Body Odor

The average a for body odor for a collective comes from an optimization against the cost of ventilation since ventilation has always been optimized against the cost for ventilation. (13) is the derivative at optimum or marginal loss against marginal cost. Dem(x_1) from Table 2.

$$a \cdot Dem(x_1) = \frac{dK(x_1)}{dx_1} \tag{13}$$

a is chosen so that the optimum outdoor air rate for body odor will be 10 and 15 l/s pers. The investment in a ventilation system with 1,25 m^3/s supply and return air (100 % outdoor air) with heat recovery in an office building with 50 rooms is 1 MSEK [2]. If half the investment depends on the outdoor air rate and the annuity is 0,074 then the cost of capital is 30 000 SEK/m^3/s, yr. The cost of operation (heat and maintenance) is 7 400 SEK/ m^3/s, yr during 2000 h/yr work hours. Then it will cost l = 37,4 SEK/ l/s yr to increase the outdoor air rate at the design of the building. The fixed investment is 500 000 / 50 room * 0,074 = 740 SEK/ room yr. The source strength of CO_2 is f = 16 l/h, pers = 0,0044 l/s pers (14).

$$K(q) = m + l \cdot q = m + \frac{l \cdot f}{c} = 740 + \frac{0{,}17}{c} \tag{14}$$

The cost in h/yr pers c = x1 * c*1. The price of an input hour is 250 SEK/h and the odor threshold for the most sensible (1,3 %) is c*1 = 50 ppm = 0,00005.

$$K(x1) = 3 + \frac{13,6}{x1} \qquad (15) \qquad\qquad \frac{dK(x1)}{dx1} = -\frac{13,6}{x1^2} \qquad (16)$$

The average demand of improvement in a collective Dem(x_1) from Table 2 is compared with the derivative of the cost in hours per pers and year in (16). This gives a for the assumed optimum outdoor air rates in Table 3.

The curves for Lossh(x1) with a log-normal distribution and a log-linear distribution are shown in Fig. 2. The normal distribution gives a less steep curve for the loss than the linear.

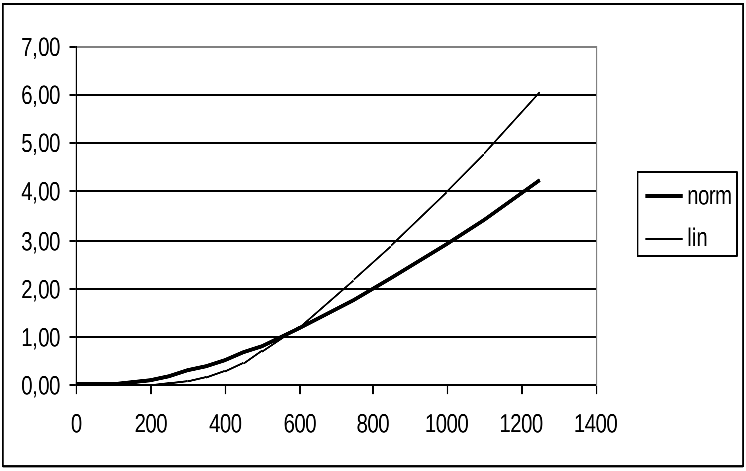

Fig. 2 Loss in hours/pers yr in body odor for log-normal distributed odor thresholds norm if the optimal outdoor air rate is 10 l/s and the loss calculated with a log-linear distribution of odor thresholds, lin as function of the CO2 concentration in ppm above outdoors.

Table 3. Dilution to threshold x1 for body odor, derivative of cost in hour/ yr pers and average demand of improvement for a collective Dem(x1) a and loss at the optimum air rate.

air rate	x1	dK(x1)/dx1	Dem(x1)	a	lossh opt
l/s				h/yr, pers	h/yr, pers
10	12	0,17	0,072	2,4	1,1
10	9	0,17	0,058	2,9	0,7
15	6	0,38	0,041	9,3	0,93

7. Determination of a in Body Odor

The assumption that the only odor in an office room is body odor is replaced with the assumption that the odor is both body and building odor. If the most sensible needs 9 dilutions not to feel body odor then he needs 12 dilutions not to feel body and building odor or 1,33 times more air. If it is normal with body and building odor and the optimum air rate still is 10 l/s pers then a must be lower or a = 2,4 in Table 3. If a is lower then the loss of production in polluted air is lower.

The optimum x_1 for body odor is between 9 and 6. If the odor is body+carpet odor in 10 l/s pers then x_1 increases 4 times to 36 - 24, Table 4. If the outdoor air rate is reduced to 3 l/s pers then x_1 increases to 120 - 80. If 10 l/s is optimal and the occupants are used to both body and building odor then the loss in b+c odor at 10 l/s is 7 h/yr pers or the introduction of carpet odor increases the loss from 1,1 h/yr to 7 h/yr pers. The increased loss is 6 h/yr or 1500 SEK/yr pers. The cost for changing the carpet [3] is estimated to 1000 SEK/yr pers. The measured loss in a work rate study [6] in b+c odor at 3 l/s pers was 72 -150 h/yr pers.

Table 4. Loss at x_1 in b+c odor at the outdoor air rate 10 and 3 l/s pers.

optimal air rate	optimal x_1	b+c 10 l/s p	b+c 3 l/s p	Lossh b+c 10 l/s p	Lossh b+c 3 l/s p
l/s pers		x_1	x_1	h/yr pers	h/yr per
10	12	36	120	7	43
10	9	36	120	9	52
15	6	24	80	15	93

8. Test Against Ventilation in Swedish Schools 1879-1994

Time series of ventilation and economic data can be used to check if the function for loss has the right form. The average CO_2 level in Stockholm schools the winter 1879 was 1600 ppm above outdoor or $x_1 = 32$. [7] quotes [8] who did measurements of the CO_2-concentration in 21 classrooms in 10 schools in Stockholm. He used the modified Pettenkofer method where the CO_2 in a known air volume was absorbed in an aqueous solution of barium sulphate. The solution was titrated with oxalic acid. The average during lessons was 1868 ppm with a standard deviation between the averages of 636 ppm.

The children are assumed to give only f = 14 l/h CO_2. This gives the average outdoor air rate 2,4 l/s pers. The marginal cost for ventilation is calculated for f = 16 l/h CO_2. The outdoor air rate 2,4 l/s pers corresponds to $x_1 = 38$ when x_1 represents the cost. Dem(32) = 0,12 according to Table 2. The average CO_2 1990 was 600 ppm $x_1 = 12$ above outdoors or 6,5 l/s pers. The x_1 for cost is $x_1 = 14$ and Dem(12) = 0,072. The GNP/pers in fixed prices has increased 14 times between 1879 and 1994. This is used as the price relation between outdoor air rate and value of production. The value of production per time unit can buy 14 times more outdoor air. (17) is two (13) in relation to each other. (17) is 1,9 ≈ 1,9.

$$\frac{Dem(x_1)}{Dem(x_2)} = \frac{0{,}12}{0{,}072} = \frac{14 \cdot 14^2}{38^2} \tag{17}$$

The increase in outdoor air rate in schools between 1879 and 1994 follows the W-F Law for a collective.

9. Conclusion

The log-normal distribution of odor thresholds gives a lower loss of production at high numbers of dilutions than the log-linear distribution. The loss is still close to the measured losses in work rate studies. The form of the loss function based on a log-normal distribution gives a good fit to data of ventilation and economic conditions from Swedish schools 1879 and 1990.

10. Discussion

The function for individual loss is derived from the Weber-Fechner law of individual odor perception. The individual loss and the log-normal distribution of odor thresholds or the percentage dissatisfied PD at first entrance give the function for loss in a collective. The test against a simple time series looks promising. It should be tested against other time series to see how the form of the function fits the series.

11. References

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