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OPTIMAL TWO PART TARIFF FOR HEAT COST ALLOCATION IN MULTIPLE UNIT DWELLINGS

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Abstract. The paper studies how the heat transfer between the dwellings increases the cost of an indoor temperature increase. The cost increase makes the households chose a lower indoor temperature that give inconveniences with a value many times higher than the positive value of individual heat cost allocation it self. A two part tariff with a division into a fixed and a variable part reduces the cost of a temperature increase. The cost should be the same as for only the heat loss to the outdoor air since heat transferred to other dwellings is not used. The cost of an increase also depends of the fixed losses in the heating system. A fixed part between 0.3 and 0.5 gives in best case an inconvenience two times higher than the positive value of heat cost allocation. The fixed part must be calculated for every dwelling separately.

Keywords: tariff, allocation, heat, cost, fixed, variable, two part tariff, heat transfer

1. Introduction

The household balances the inconvenience of a low temperature against the cost for energy. The household should pay for the energy used to produce the heat that goes out to the outdoor air as a result of a temperature increase in the dwelling. The temperature increase is close to the used temperature. The heat is no longer possible to use when the temperature has reached the outdoor temperature. Heat transferred to other dwellings goes to a lower temperature and will be useful for heating in the receiving dwelling.

The increase of the marginal cost of temperature from heat transfer in the walls between the dwellings was shown by [1]. The demand curve for indoor temperature in Swedish multiple unit dwellings was determined in [2] as a straight line and in Swedish single unit dwellings in [3] and [4]. If individual heat cost allocation is used then the indoor temperature will be reduced by the heat transfer between the dwellings since the marginal cost of temperature is increased in [5], [6] and [7].

It is known from [8] that a higher variable part in a two part tariff reduces the heat use in a building with MU-dwellings.

2. Problem formulation

The household hh chooses the indoor temperature that minimises the sum of the cost of cold and the cost of heat in equation (1).

$$\frac{k\cdot DI'}{2}\cdot\left(t^{*}-t\right)^{2}+CH_{o}(t) \tag{1}$$

where k - a constant 1/°C^2, DI' - the disposable income per hour EUR/hh h, t* - the highest demanded indoor temperature °C, t - the indoor temperature °C, and $CH_o(t)$ - the cost of heat to the outdoor air as function of the indoor temperature EUR/h hh. It has its optimum at t according to equation (2).

$$-k\cdot DI'\cdot\left(t^{*}-t\right)+\frac{dCH_{o}(t)}{dt}=0 \tag{2}$$

The second term in equation (2) or the cost of a temperature change for the hh should be measured and calculated according to equation (3). The cost of heat at a temperature change is ideally the cost for the energy used to produce the heat needed for the temperature change. The cost of a change in indoor temperature for the heat loss out can be written according to equation (3)

$$\frac{\partial CH_{o}(t)}{\partial t}=\frac{\partial CH}{\partial Q}\cdot\frac{\partial Q}{\partial t}\cdot\frac{\partial Q_{o}}{\partial Q} \tag{3}$$

where the first term on the right side is the marginal cost of heat or the price of heat EUR/MWh from the heating system. It is calculated. The second term is the measured heat demand per degree indoor temperature W/°C. A heat meter shows heat use kWh over the period between readings. The average heat power W during the period is heat use divided by the length of the period. The third term corrects the measured heat Q at a temperature change to the heat loss to the outdoor air.

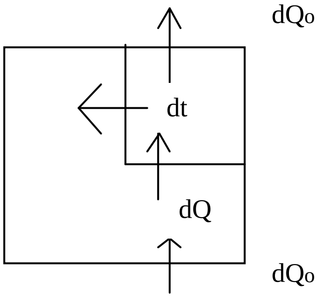

Fig. 1. Heated building with a dwelling.

If the household in (Fig. 1) increases the heat power to the dwelling with dQ then the indoor temperature will increase dt and the heat leaving the building increases with dQ_o and an extra dQ_o must enter the building via the heating system. The heat from the dwelling to other dwellings replaces heat from the heating system in the other dwellings. The household should pay for dQ_o. The heat meter in the dwelling shows dQ but if it is corrected then dQ can be used as a basis for payment.

If heat is lost to another dwelling the heat will replace heat for heating in the receiving dwelling. It is only a transfer of values. If the indoor temperature is to low then the inconvenience is a real loss.

3. Price of heat

The price of heat p_h EUR/MWh into the dwelling follows equation (4).

$$\frac{\partial CH}{\partial Q} = p_h \tag{4}$$

Ideally it is the cost for the energy that is used to make the heat. Energy in the heating system goes to losses that do not depend on the heat use in the dwelling. The price can be calculated with the cost of energy as a function of heat use or with a division in a fixed and a variable part.

The price of energy p_E EUR/MWh comes from equation (5).

$$p_E = \frac{Cost}{E} \tag{5}$$

If the energy is oil then E = m H where m is the mass of oil kg used during the period and H is the heat value MWh/kg. If the energy is district heating then E is the quantity according to the district heat meter in the building during the period. The cost is the variable cost for energy. It may include a cost for circulation of the district heating water.

3. 1 Function

The cost of energy as function of delivered heat CH(Q) during a period may follow equation (6).

$$CH(Q) = p_E \cdot \left(\frac{Q}{\eta} + E_{fix} \right) \tag{6}$$

where η - the combustion efficiency and E_{fix} - the energy used to cover the fixed losses MWh/period.

The price of heat p_h comes from a differentiation of equation (6) with respect to Q and gives equation (7).

$$p_h = \frac{p_E}{\eta} \tag{7}$$

3. 2 Fixed and variable part I

The variable part one $(1-\Phi_I)$ in (Fig. 2) indicates how big part of the used energy that is delivered to the hh:s. The rest is the fixed part one Φ_I from equation (8). The hot water is heated separately.

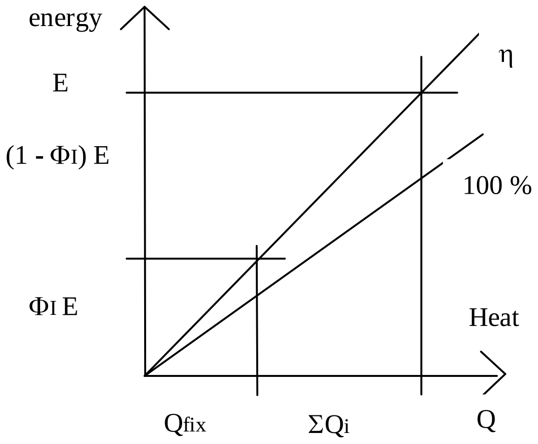

Fig. 2. Energy use as function of heat use Q during a period

where ΣQ_i - sum of measured heat from the radiators MWh, Q_{fix} - fixed losses from boiler, from pipes both outside and inside the building and heat for heating of common space such as stairs or all heat that isn't controlled by the households MWh. The cost for the variable part of the energy is distributed after the measured heat.

$$\Phi_I = \frac{Q_{fix}}{\Sigma Q_i + Q_{fix}} \tag{8}$$

If the heat from the radiators ΣQ_i are measured in heat units then the price of heat, p_h comes from equation (9).

$$p_h = \frac{p_E \cdot E \cdot (1-\Phi_I)}{\Sigma Q_i} \qquad (9)$$

If the heat cost allocaters gives “allocation units” au, then the price of an allocation unit can be calculated if ΣQ_i is replaced with the sum of allocation unit’s Σau_i. An “allocation unit” au, is direct proportional to a heat unit. If allocation units are measured then it will not be possible to determine the fixed losses from measurements. To estimate the fixed loss minimum one extra heat meter is necessary.

4. Heat for a temperature change

According to physics the heat demand for a dwelling follows equation (10).

$$Q = (\Sigma UA + q \cdot \rho \cdot c_p) \cdot (t - t_o) + \sum_i \Sigma U_i \cdot (t - t_i) - Q_{free} \quad (10)$$

where ΣUA - the sum of heat transmission to the outdoor air W/°C, A - area m^2, U - heat transfer coefficient $W/m^{2}°C$, q - outdoor air rate for ventilation m^3/s, ρ - density of air kg/m^3, c_p - specific heat of air kJ/kg°C, ΣU_i - the sum of specific heat transfer between the dwelling and dwelling i trough the common walls W/°C, t - indoor temperature in the dwelling °C, t_i - indoor temperature in the surrounding dwelling i °C, Q_{free} – free heat from the sun, light, refrigerator, persons and pipes with no control in the dwelling W.

Equation (10) can be written in a shorter form equation (11).

$$Q = \Sigma U_o \cdot (t - t_o) + \sum_i \Sigma U_i \cdot (t - t_i) - Q_{free} \qquad (11)$$

where ΣU_o - the specific heat demand to the outdoor air including transmission and ventilation W/°C.

The demand for heat in a dwelling from the heating system starts at the balance temperature t_b. If the heaters are shut of the indoor temperature will fall to t_b since heat from other dwellings is transferred trough the walls.

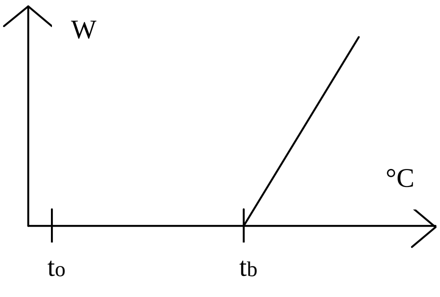

Fig. 3. Heat demand in a MU-dwelling as a function of the indoor temperature at the outdoor temperature t_o. Balance temperature t_b

Differentiate equation (11) with respect to t gives the heat needed to increase the indoor temperature dt in equation (12).

$$\frac{\partial Q}{\partial t} = \Sigma U_o + \sum_i \Sigma U_i \qquad (12)$$

4. 1 Correction for heat transfer

If an extra unit of heat dQ is supplied to a dwelling it will increase the indoor temperature and a fraction of the supplied heat dQ_o will go to the outdoor air and the rest to the surrounding dwellings equation (13) or the variable part two ($1-\Phi_{II}$)

$$\frac{\partial Q_o}{\partial Q} = \frac{\Sigma U_o}{\Sigma U_o + \sum_i \Sigma U_i} = (1 - \Phi_{II}) \qquad (13)$$

5. Demanded indoor temperature

Equations (2), (3), (9), (12) and (13) gives the demanded indoor temperature equation (14). If the factors for correction Φ_I and Φ_{II} are ideal then the the cost of cold at the indoor temperature t is optimized against the cost for heat leaving the dwelling to the outdoor air.

$$t = t^* - \frac{p_E \cdot E \cdot (\Sigma U_o + \Sigma\Sigma U_i) \cdot (1 - \Phi_I) \cdot (1 - \Phi_{II})}{\Sigma Q_i \cdot k \cdot DI'} \qquad (14)$$

6. Value of errors

If the factors for correction Φ_I and Φ_{II} in equation (14) not are ideal then the result will be an extra inconvenience A EUR/hh yr. For example if the cost for a temperature increase is to high then the indoor temperature will be to low and the inconvenience of having a to low indoor temperature will be to high.

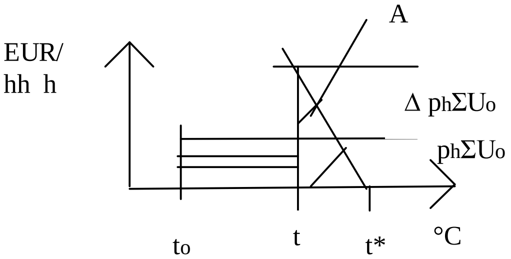

Fig. 4. Value of an extra inconvenience A

The value of the extra inconvenience is the difference between an increased cost of cold and a reduced cost of heat or the area of the triangle A in (Fig. 4) and equation (15).

$$A = \frac{(\Delta \cdot p_h \cdot \Sigma U_o)^2}{2 \cdot k \cdot DI'} \cdot \tau \qquad (15)$$

where Δ - the error in the ideal cost of a temperature increase, $p_h \Sigma U_o$ - marginal cost for heat loss to outdoor air with a price of heat corresponding to cost of energy EUR/°C h, τ – length of period h.

6. 1 Example

If the specific heat demand for heat out ΣU_o = 100 W/°C, the sum of specific heat transfer to other dwellings $\Sigma\Sigma U_i$ = 300 W/°C, k = 0.004 1/°C^2, DI = 4000 EUR/hh yr or DI´ = 0.5 EUR/hh h and τ = 230 days x 24 h/day. The ideal cost of a temperature increase is the cost for the heat loss to the outdoor air at Φ_{II} = 0.75 and the price of heat is the cost of energy used to produce the heat at Φ_I = 0.3. The fixed loss is then 30 %. The extra inconvenience A follows Table 1.

Table 1. Correction of the price of heat (1 - ΦI). Correction of the heat transfer with other dwellings (1 - ΦII). Error Δ %. Indoor temperature t. Value of extra inconvenience A.

1-ΦI	1-ΦII	Error Δ %	ph = 15 EUR/ MWh	A EUR/ hh yr	ph = 45 EUR/ MWh	A EUR/ hh yr
			t °C		t °C	
0.7	0.2	-20	21.05	0.1	19.85	1.1
0.7	0.25	0	20.90	0.0	19.40	0.0
0.7	0.3	20	20.75	0.1	18.95	1.1
0.7	0.4	60	20.45	1.1	18.05	10
0.7	0.5	100	20.15	3.1	17.15	28
0.7	0.7	180	19.55	10	15.35	91
0.7	1	300	18.65	28	12.65	252
0.8	1	360	18.22	40	11.36	356
0.9	1	410	17.79	53	10.08	480
1	1	470	17.36	69	8.79	621

6. 2 Value of error in relation to economy

The maximum value of heat cost allocation or the reduction of costs RC at a change from a collective to individual indoor temperatures is given in equation (16). It is presented in another paper at this conference. To get the reduction of costs per year multiply with the length of the heating season τ h/year.

$$RC = RC°\left(t°_i\right) \cdot k \cdot DI' \cdot s^2 \qquad (16)$$

The dimensionless reduction of costs, is approximative RC° = 0.5 at p_h = 45 EUR/MWh and the reduction of costs RC = 15 EUR/dw yr minus the cost for reading and administration ca 10 EUR/dw yr. There is 5 EUR/dw yr left to cover the investment in the meters. The value of the extra inconvenience A can not be more than 2-3 EUR/dw yr at p_h = 45 EUR/MWh. This corresponds to an error Δ of 35 %. It can be solved from equation (15).

7. Insufficient compensation

With no correction for heat transfer (1 – Φ_{II}) = 1 the marginal cost is 300 % to high according to Table 1. The correction for fixed losses in the heating system is then removed until (1 -Φ_I) = 1. Then the marginal cost is 470 % to high. The increased marginal cost reduces the indoor temperature and the extra inconvenience increases with the square of the error.

If the error is seen as a change in price of heat then p_h = 15 EUR/MWh would increase to 60 EUR/MWh as a result of heat transfer in the walls. If both Φ_I = 0 and Φ_{II} = 0 then the price of heat would increase to 85 EUR/MWh. At the higher price p_h = 45 EUR/MWh it would increase to 180 EUR/MWh at Φ_{II} = 0 and to 260 EUR/MWh at Φ_I = 0 and Φ_{II} = 0.

7.1 A fixed part Φ_{II} for all dwellings

Two dwellings in a building with 45 dwellings from [9] are used as example. Heat transfer data in Table 2 for a dwelling in the perimeter of the building and in Table 3 for a dwelling in the centre of the building.

Table 2. Heat transfer data for a dwelling in the perimeter under roof and at short wall. Orientation out is against the outdoor air and in against other dwellings.

Part	Area, m^2	U-value W/°C, m^2	W/°C	Orient
Wall	50	1.4	70	out
Wall	37	3	110	in
Window	6	2.6	16	out
Roof	40	1.0	40	out
Floor	40	2	80	in
Door	2	2.4	5	in

Heat transfer out = 126 W/°C. Heat transfer in = 195 W/°C. Heat for ventilation 0.030 m^3/s x 1.2 kg/m^3 x 1 kJ/kg C = 36 W/°C. Total heat out 126 + 36 = 162 W/°C. The variable part (1-Φ_{II}) according equation (13) is 162 / (162 + 195) = 0.45.

Table 3. Heat transfer data for a dwelling in the centre. Orientation out is against the outdoor air and in against other dwellings.

Part	Area, m^2	U-value W/°C, m^2	W/°C	Orient
Wall	23	1.4	32	out
Wall	60	3	180	in
Window	6	2.6	16	out
Roof	40	2	80	in
Floor	40	2	80	in
Door	2	2.4	5	in

Heat transfer out = 48 W/°C. Heat transfer in = 345 W/°C. Heat for ventilation 36 W/°C. Total heat out 48 + 36 = 84 W/°C. The variable part (1-Φ_{II}) according to equation (13) should be 84 / (84 + 345) = 0.20.

The variable part for the dwelling in the perimeter of the building in Table 2 is 1 – 0.55 = 0.45. The variable part for the dwelling in the centre of the building in Table

3 is 1 - 0.8 = 0.2. The average of the variable parts is (0.45 + 0.2) / 2 = 0.33.

The average variable part for both dwellings will give the dwelling in the perimeter 0.33 / 0.45 = 73 % or an error of -27 %. The average variable part for both dwellings will give the centre dwelling 0.33 / 0.20 = 165 % or an error of 65 %. The centre dwelling will get an error that is to high compare to maximum 35 %.

7. 2 A fixed part Φ_I for all years

If the fixed losses in the heating system are constant from year to year and the same fixed part Φ_I is used every year then the price of heat from equation (9) will be different every year. If a building with boiler uses 400 MWh/yr energy at the price 100 units/MWh and the fixed loss is 120 MWh/yr then the delivered heat to the households is 280 MWh/yr. The fixed part according to equation (8) is 0.3.

If the building during a cold year uses 440 MWh/yr energy and the fixed loss still is 120 MWh/yr then 320 MWh/yr is delivered to the household. If equation (9) is used the price of heat will be 96.25 units/MWh. This is an error of - 3.75 %. If the building during a mild year uses 360 MWh/yr energy and the fixed loss still is 120 MWh/yr then 240 MWh/yr is delivered to the households. If equation (9) is used the error will be 5 %.

If the division in a fixed and a variable part is close to the fixed losses then the error in the price of heat from the yearly variation in heat use is acceptable.

7.3 A combined fixed part 0.3 – 0.5

According to [8] a fixed part between 0.3 and 0.5 is used in Germany. If the building has a boiler and a fixed loss of 30 % and the fixed part 0.3 is used then the error according to Table 1 is 300 %. If the building has district heating and no fixed loss then the error with the fixed part 0.3 is 180 %. The value of the error is between 91 and 252 EUR/hh yr. It is many times the positive value of heat cost allocation.

If the building has district heating, no fixed losses and a fixed part 0.5. Then the error is 100 % and the value of the error 28 EUR/hh yr, witch is still twice the value of heat cost allocation if the cost for reading and administration is neglected.

The combined variable part for the dwelling and the heating system in Table 1 is (1 - ΦI)*(1 - ΦII) = 0.7 * 0.25 = 0.175 and the fixed part 0.825. In a building with district heating the fixed losses are smaller so the combined variable part is closer to 0.25 or a fixed part 0.75. The fixed part used in Germany 0.3 – 0.5 is to low compared to 0.75 – 0.825.

8. Heat to a dwelling

The measured heat to a dwelling will have a variation depending on the temperature in the dwelling and depending on the temperature in the surrounding dwellings. With correction the hh will pay the variable part according to equation (17) during the period τ.

$$p_h \cdot (Q \cdot \tau) \cdot \frac{dQ_o}{dQ} \qquad (17)$$

8.1 Heat to outdoor air

The variation in the heat loss to the outdoor air depends on the variation in indoor temperature. The indoor temperature has a standard deviation of s = 1.3°C. Specific heat transfer out ΣU_o = 100 W/°C. The average dwelling is supposed to have 21°C indoors and with 5°C outdoors the heat loss out is 1600 W. At 2 standard deviations there will be 2.3 % of the households who loses more than 1860 W and 2.3 % who loses less than 1340 W.

8.2 Heat exchange with other dwellings

The dwelling in (Fig. 5) has heat exchanging areas against the outdoor air and against four other dwellings and an outdoor air rate for ventilation.

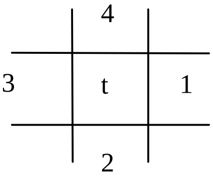

Fig. 5. A dwelling in heat exchange with four surrounding dwellings

The average temperature in the building is t_m and the indoor temperatures t in the dwellings have a standard deviation, s. If the specific heat transfer is approximately the same between the dwelling and all the surrounding dwellings, then the net heat transfer can be expressed with the average temperature in the surrounding dwellings t_s. The average of all t_s is t_m. The average of the temperature in four dwellings, t_s has a standard deviation according to equation (18)

$$\frac{\sqrt{4 \cdot s^2}}{4} = \frac{s}{2} \qquad (18)$$

If a hh has the average indoor temperature t_m then the standard deviation of the net heat transfer with the four surrounding dwellings follows equation (19) with $\Sigma\Sigma U_i$ = 300 W/°C.

$$\Sigma\Sigma U_i \cdot \left(\frac{s}{2}\right) = 300\frac{W}{°C} \cdot \frac{1.3°C}{2} = 195W \qquad (19)$$

If a household has the indoor temperature 21°C and the outdoor temperature is 5°C then the heat loss to the outdoor air is 1600 W and there will be 2.3 % of the households with 21°C indoor who uses more than 1990 W and 2.3 % who uses less than 1210 W depending on the temperature in surrounding dwellings.

8.3 Total variation in measured heat to a dwelling

In equation (11) the first term is the heat loss to the outdoor air. t and t_o is chosen so that $Q \geq 0$. If the specific heat transfer W/°C with the surrounding dwellings is approximately the same for all surrounding dwellings and if Q_{free} is neglected then Q is given by equation (20).

$$Q = \Sigma U_o \cdot (t - t_o) + \sum_i \Sigma U_i \cdot (t - t_s) \tag{20}$$

where t_s is the average temperature in the four surrounding dwellings. The standard deviation for Q will follow equation (21).

$$\sqrt{(\Sigma U_o)^2 \cdot s^2 + (\Sigma\Sigma U_i)^2 \cdot s^2 + (\Sigma\Sigma U_i)^2 \cdot \left(\frac{s}{2}\right)^2} \tag{21}$$

where ΣU_o = 100 W/°C, $\Sigma\Sigma U_i$ = 300 W/°C and the standard deviation for indoor temperature s = 1.3°C. This gives the standard deviation for heat power 455 W. The average dwelling is supposed to have 21°C indoors and with 5°C outdoors the heat loss out is 1600 W then there will be 2.3 % of the households who uses more than 2510 W and 2.3 % who uses less than 690 W. The balance temperature t_b in (Fig. 3) is 17°C. The hh at 2 standard deviations has an indoor temperature of 18.4°C. This means that the household do not need to heat the dwelling at outdoor temperatures over 10.6°C.

Table 4. Heat use in a dwelling. Average ± 2 standard deviations s.

	Heat (W)	Relative
Heat out	1600 ± 260	1 ± 0.16
Heat exchange	1600 ± 390	1 ± 0.24
Total	1600 ± 910	1 ± 0.57

Heat out is the variation in heat exchange with the outdoor air from the normal variation in the preferred indoor temperature in the dwelling. Heat exchange is variation in heat use in the dwelling from different temperatures in surrounding dwellings if the temperature in the dwelling is 21°C. The total variation will be high since the heat transfer with surrounding dwellings is high. This makes the heat use very sensible to the indoor temperature in the dwelling.

9. Conclusions

The heat transfer between the MU-dwellings increases the price of indoor temperature. This makes the households chose a to low indoor temperature. This can be compensated with a two part tariff. It has a fixed part and a variable part.

The fixed part used for all dwellings in a building to day is too small. It is necessary to calculate a fixed part for every dwelling from the heat transfer data of the walls against the outdoor air and against other dwellings. This will increase the value of individual heat cost allocation after measured heat quantity.

The cost of a temperature increase can not be more than 30 – 40 % higher than the cost for the heat loss to the outdoor air. Else the value of the inconvenience from a low indoor temperature will be to high in relation to the reduced costs with individual heat cost allocation.

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