INDOOR TEMPERATURE AS A COLLECTIVE GOODS

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ABSTRACT

A collective goods can be consumed by many without reducing the available quantity for others. A theoretical demand curve for a collective indoor temperature is derived. The collective indoor temperature goes toward the highest demanded indoor temperature at low prices of heat or at high disposable incomes. The measured average of collective temperatures in Sweden 1952, 65, 82-85 and 92 are app. 0.5ºC higher than the theoretical. The demand curve can also be used to calculate how balancing accuracy of hydronic heating systems affects the average indoor temperature.

INTRODUCTION

The first men on earth needed food, drink and clothes. If one person has eaten a piece of food it is no longer available for others. These goods have got the name private goods in economics. When fire was taken into use indoors it raised the indoor temperature above the outdoor temperature. Indoor temperature can be used by many without reducing it for others. Goods that can be consumed without reducing the available quantity for others have got the name collective or public goods in economics.

Indoor temperature is consumed as a collective goods in multiple unit dwellings where the supply water temperature is controlled after the outdoor temperature.

A theory for the choice of the best quantity of collective or public goods was developed by Erik Lindahl 1919 according to Bergstrom, 2005. Ståhl, 1975 called heating a semi-collective goods, but heating in Swedish dwellings with district heating is more a collective goods since the main control is the pre-shunting. He divided the marginal cost for indoor temperature by the number of dwellings in a building, n. The theory for collective goods was explained by Bohm, 1977. Friedman, 1986 used the theory for public goods to find the best common indoor temperature in a building with two dwellings.

The demand curve for indoor temperature in Swedish multiple unit dwellings was determined by Jönsson, 1997 as a straight line and in Swedish single unit dwellings by Jönsson, 2004. Heat cost allocation in homes with individual control and individual heat metering, individual temperature metering and collective allocation was described by Jönsson, 2005.

HYDRONIC HEATING WITH PRESHUNTED SUPPLY WATER AND COLLECTIVE HEAT COST ALLOCATION

Hydronic heating normally uses warm water to carry the heat from the boiler to the radiators in the dwellings. The supply water temperature is controlled in relation to the outdoor temperature so the indoor temperature is kept constant in time. The radiators are chosen so that the indoor temperature should be the same in all dwellings, figure 1.

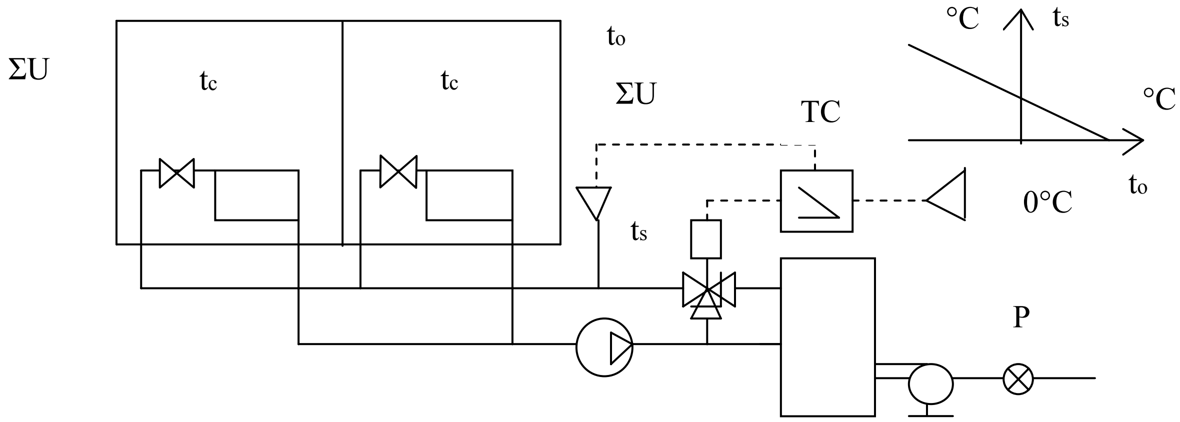

Figure 1. Diagram of hydronic heating system with preshunting after outdoor temperature, t_o

The temperature of the supply water and thereby the temperature in all dwellings is determined by the settings of the supply water temperature controller, TC. It is only the owner or the owners representative who has access to the controller. Every radiator has a radiator valve to shut off or to reduce the supply water flow. Swedish dwellings built or renovated after 1975 have thermostatic radiator valves. The households must contact the owner and demand a raise of the indoor temperature. The heat cost for the whole building is distributed among the households after dwelling area. The quantity of heat or energy is measured at P.

CHOICE OF INDOOR TEMPERATURE IN A DWELLING

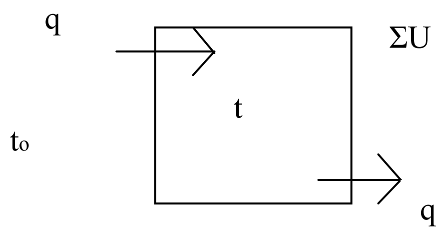

Figure 2. Simplified building with one dwelling

The heat cost per time unit SEK/h at indoor temperature t will bee equation (1 a).

$$p_h \sum U \cdot (t - t_o) \qquad \Sigma U = \sum_i Uil \cdot Ail + ql \cdot \rho \cdot c_p \qquad (1\ a, b)$$

$p_h \Sigma U$ is the heat cost per time and temperature unit SEK/h°C for a household. The cost of heat is represented bye the horizontally marked rectangle in figure 3 the upper side of the rectangle is the supply curve. When the indoor temperature is below t* then the household will feel cold and the feeling of cold is valued to the area of the triangle in figure 3.

$$\frac{DI' \cdot k}{2}\left(t^* - t\right)^2 \qquad (2)$$

t* is the starting point of the demand curve. DI' is the disposable income for the household SEK/hh h. k is a constant from regression, $1/°C^2$. The triangle is the demand curve and the upper side of the rectangle is the supply curve. If the indoor temperature is raised over t* then the temperature t^o will be reached where a cost of "to warm" will occur. The cost of "to warm" is neglected in this paper.

$$DI' \cdot k \cdot (t^* - t_d) = p_h \cdot \sum U \qquad (3)$$

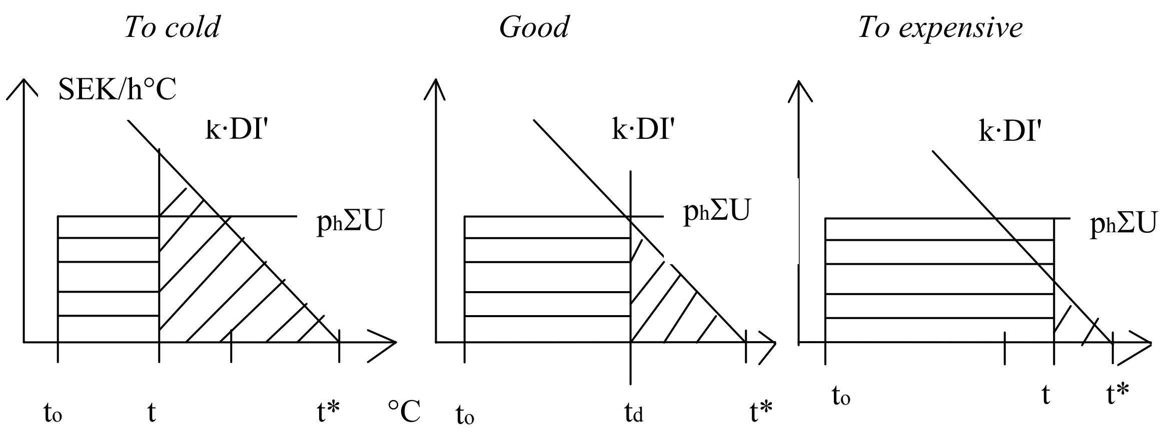

Figure 3. Demand curve and supply curve for a household gives demanded indoor temperature, t_d in a household. Cost of cold is diagonally and cost of heat is horizontally marked, Jönsson, 2005

The sum of cost of heat and the cost of cold is minimised when a household chooses the demanded indoor temperature, t_d from equation (3) and figure 3.

U_j specific heat loss to outdoor air from dwelling j W/°C
A_{ij} area i against outdoor air in dwelling j m^2
U_{ij} heat transfer coefficient of area i in dwelling j $W/m^2°C$
q_j outdoor air rate in dwelling j m^3/s
ρ density of air kg/m^3
c_p specific heat of air kJ/kg°C
p_h price of heat SEK/Wh
SEK Swedish Kronor, 8 SEK = 1 USD

DEMANDED INDOOR TEMPERATURE

The demanded indoor temperature in Swedish households was measured by the Swedish Institute of Building Research. During the years 1982-85 the indoor temperature in 150 different SU- and 150 different MU-dwellings were measured every year and 1992 the indoor temperature in 600 SU- and 600 MU-dwellings were measured. The dwellings were randomly distributed over the whole country so they are national averages of the indoor temperatures in Sweden. The measurements from 1952 and 1965 are only from buildings in one city. The value from 1971 is an estimate.

The MK/DI are calculated from price of oil, oil consumption, from Statistics Sweden and the assumption that the oil consumption should increase 7 % per year if the indoor temperature were increased 1ºC. The result is given in figure 4 from Jönsson 2004.

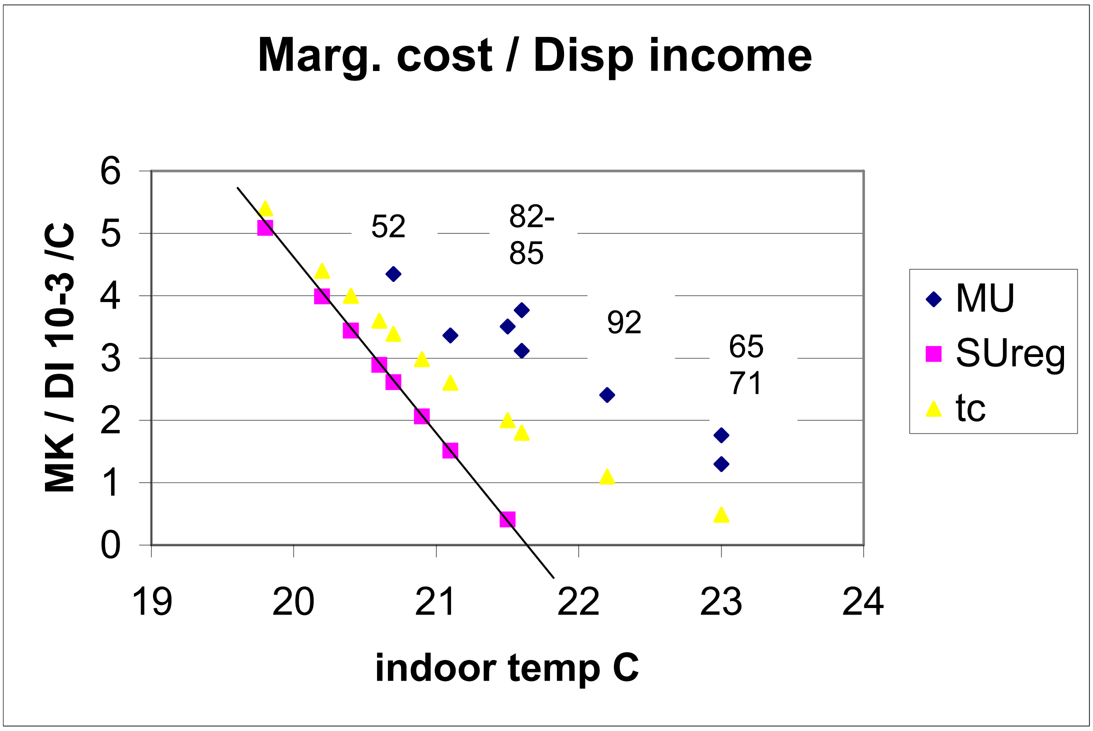

Figure 4. Marginal cost, MK for indoor temperature per year in a single unit, SU resp. in a multiple unit dwelling, MU divided by the disposable income, DI for the household in the dwelling against demanded indoor temperature. Indoor temperature in SU-dwelling according to the line of regression, SUreg. t_c is the collective temperature

MK/DI is correlated against the demanded indoor temperature, t_d in Single unit dwellings in equation (4).

$$MK/DI = 2.75\ 10^{-3}\ (21.65 - t) \qquad r = 0.95 \qquad (4)$$

CHOICE OF A COLLECTIVE TEMPERATURE

The collective temperature, t_c is the common temperature in all dwellings in a building. It is chosen to minimize the sum of the total cost of heat for all households in the building and the total cost of cold for all households in the building. In the calculation of the total cost of cold all households are assumed to have the same inclination of the demand curve as the average and to have the same heating cost as the average heating cost all households have the average disposable income and lives in average dwellings. If the starting points for the cost of cold t^*_1 to t^*_n belongs to a normal distribution $f(t^*_m, s^*)$ with frequency f(t) and distribution F(t) functions with t indoor temperature, s* standard deviation, t^*_m average t* equations (5 a, b).

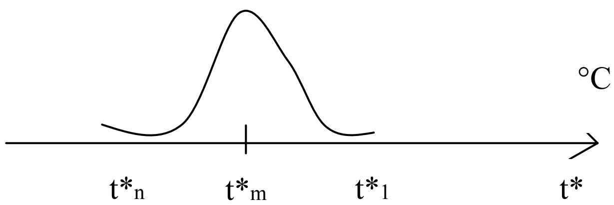

Figure 5. Frequency curve f(t) for t*i who belongs to a normal distribution

$$f(t) = \frac{1}{s^* \sqrt{2\pi}} e^{-\frac{(t-t^*m)}{2s^{*2}}} \qquad F(t) = \frac{1}{s^* \sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(t-t^*m)}{2s^{*2}}} \qquad (5\text{ a, b})$$

The total cost of cold, TCC or the sum of all cost of colds for all n households in a building at the indoor temperature t is equation (6 - 8). If the indoor temperature in a household is higher than t^*_i for household i there is no cost of cold in household i. The cost of "to warm" is neglected in this paper.

If $t > t^*_1$ $\quad 0 \quad$ (6)

If $t^*_1 > t > t^*_2$ $\quad \frac{DI' \cdot k}{2}\left(t^*_1 - t\right)^2 \quad$ (7)

If $t^*_2 > t > t^*_3$ $\quad \frac{DI' \cdot k}{2}\left(\left(t^*_1 - t\right)^2 + \left(t^*_2 - t\right)^2\right) \quad$ etc $\quad$ (8)

Total cost of heat for n identical dwellings or the cost of indoor temperature at t is:

$$n \cdot p_h \sum U(t - t_o) \qquad (9)$$

The minimum of the total cost of cold and the total cost of heat is found by differentiation of the total cost of cold and the total cost of heat with regard to t. The equality of the differentials gives t_c according to equation (10).

$$DI' \cdot k \sum_i \left(t^*_i - t_c\right) = n \cdot p_h \sum U \qquad (10)$$

Figure 6 shows a vertical addition of the demand curves for the separate households. The collective temperature t_c will be between t^*_3 and t^*_4 and i = 3.

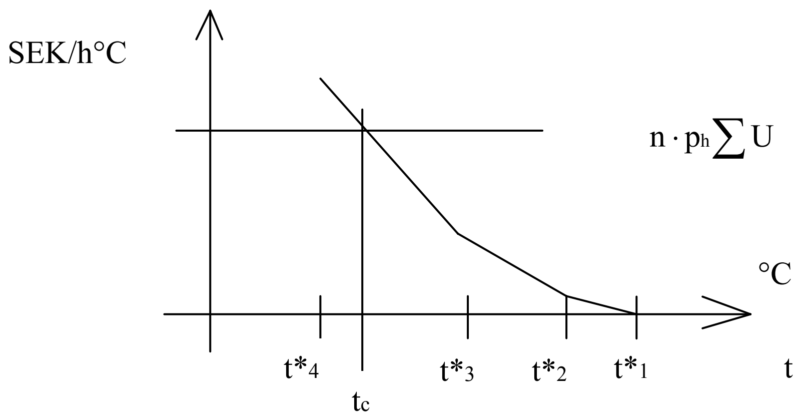

Figure 6. The differential of the total cost of cold equals the differential of the cost of heating for the building at t_c. The area under the polygon is the total cost of cold

The inclination of the differential of the total cost of cold with regard to t between t^*_i and t^*_{i+1} is:

$$-i\ DI'\ k \qquad (11)$$

If the inclination of the differential of the total cost of cold is divided by n then

$$\frac{d\ DTCC}{dt} = -\frac{i \cdot DI' \cdot k}{n} \qquad (12)$$

t^*_i belongs to a normal distribution according to figure 5 and n is so big that the inclination of DTCC can be regarded as continual, then:

$$\frac{dDTCC}{dt} = -\left(1 - F(t^*_m, s^*)\right) \cdot DI' \cdot k \qquad (13)$$

$$DTCC = DI' \cdot k \int_{\infty}^{tc} (1 - F(t^*_m, s^*))dt \qquad (14)$$

The distribution (1- F) was integrated with the numerical trapeze method and the result is shown in figure 7 as function of the indoor temperature in standard deviations under and over $t^*_m = 21.65$ºC.

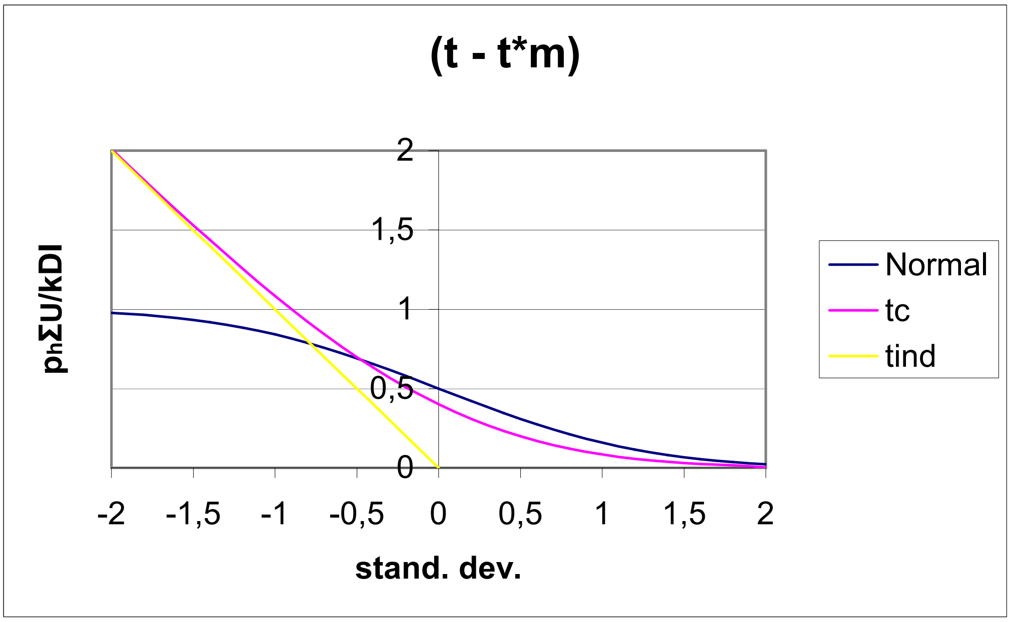

Figure 7. Normal distribution, ($t - t^*_m$) in standard deviations for a collective temperature, t_c and for individual indoor temperature metering, t_{ind}.

t_{ind} gives the average indoor temperature if heat was sold after indoor temperature $p_h \Sigma U(t - t_o)$ to every household with DI' k. The vertical axis is $p_h \Sigma U/DI'k$, in standard deviations. The normal distribution shows the distribution of the individual t^*_i .

t_c is the collective temperature at $p_h\Sigma U/DI'\,k$. If heat is almost free then the indoor temperature goes towards t^*_m if individual indoor temperature metering is used and anganist t^*_1 the highest t^* if a collective temperature is used.

EXAMPLE WITH THREE DWELLINGS

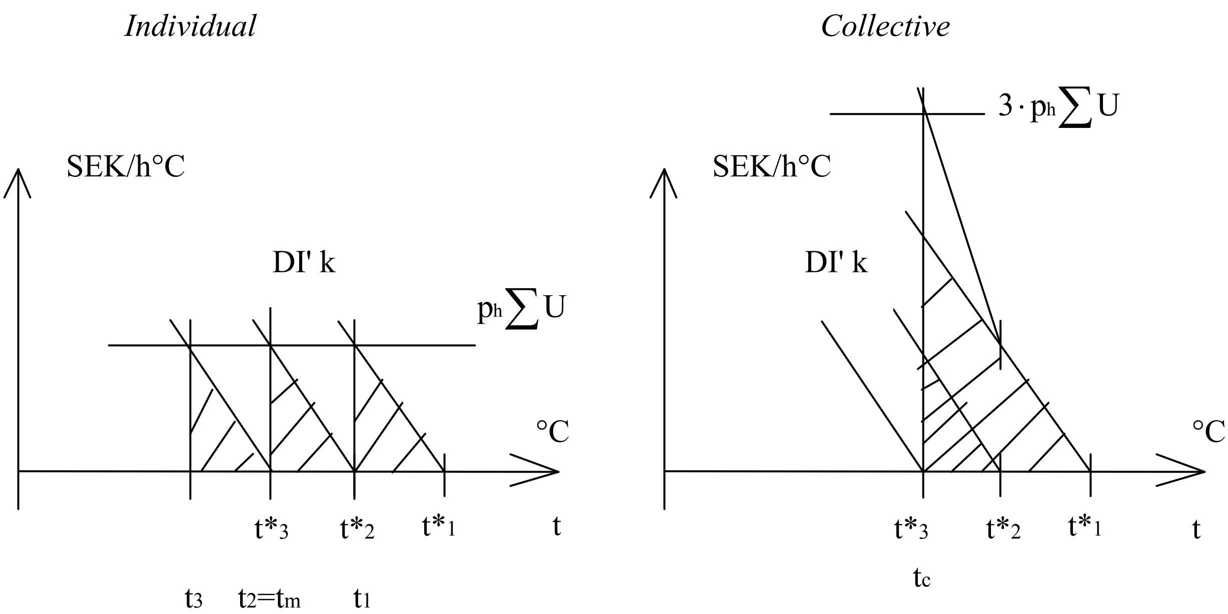

Figure 8. Choice of indoor temperature in three households with heat cost allocation after indoor temperature, left and with a common collective temperature t_c, right

If the three households who lives in identical dwellings in figure 8 have individual temperature control and pay after indoor temperature they will all chose the indoor temperature where supply meets demand for the household. There will be three separate indoor temperatures, t_1, t_2 and t_3. The average indoor temperature, t_m will coincide with t_2.

If the tree households in identical dwellings should chose a common indoor temperature then the marginal total cost of cold would bee equal to three times the marginal cost of heat for one dwelling. In figure 3 t_c is equal to t_2. The total cost of cold is higher than with individual control and it is household 1 and 2 who carries it. Household 3 with its low demand of indoor temperature do not have a cost of cold at all. Household 3 has an indoor temperature that would have been to expensive for household 3 if the temperature control had been individual.

CHOICE OF A COLLECTIVE TEMPERATURE DISTRIBUTION

Even if the heating system is built and controlled to give the same indoor temperature in all dwellings in a building there will not be the same temperature in all dwellings due to for instance insufficient balancing, non uniform insulation etc. The temperatures in the dwellings is assumed to follow a normal distribution with an average temperature, t_m and a standard deviation, s.

The starting points of the individual demand curves t^*_i still belongs to a normal distribution with an average temperature, t^*_m and a standard deviation s^* equation (5 b)

$$t_m - t^*_m \tag{15}$$

Then the difference in equation (15) belongs to a normal distribution with a standard deviation (s*^2 + s^2)^0.5. The distribution in figure 9 shows the share of households sh who have an indoor temperature lower than t^*_i. Households with a lower indoor temperature than t^*_i have a cost of cold.

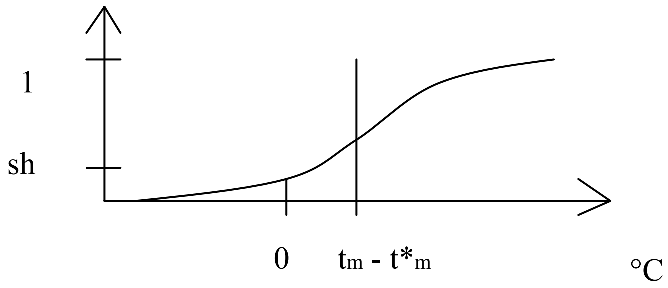

Figure 9. Indoor temperature in dwellings t minus t* for the households. t_m is the average indoor temperature in the dwellings. Normal distribution (s*^2 + s^2)^0.5

The difference (t_m - t^*_m) corresponds to the difference t_c - t^*_m in figure 7 since the standard deviation for t_m - t_m* is bigger than for t* then t_m will be bigger than t_c or $t_m>t_c$. To put people with different t* in dwellings with different temperatures increases the variation and increases the height of the sum of lines in figure 6. Then the average, t_m must be increased.

EXAMPLE

The t* is normally distributed with t^*_m = 21.65°C and s* = 1.3°C. s* is based on measurements in Norlen and Andersson 1993. The temperature in the dwellings in a building with the heating system in figure 1 follows a normal distribution with s = 0.8°C according to an estimate from measurements in Holgersson and Norlen 1983.

The (t_m - t^*_m) will have the standard deviation (1.3^2 + 0.8^2)^0.5 = 1.6°C

If the indoor temperature with individual temperature measurements in the dwellings would have been 21.0°C and if the same MK/DI relation is used with collective heat cost allocation then the temperature in the MU should have been 21.6°C according to figure 7 and it was 22.2°C.

If the balancing of the heating system is perfect then s = 0 and (1.3^2 + 0^2)^0.5 = 1.3°C which according to figure 7 gives t_c = 21.4°C. A perfect balancing would reduce the average indoor temperature with 21.6 - 21.4 = 0.2°C. If the balancing of the heating system is so bad that s =1.6 and (1.3^2 + 1.6^2)^0.5 = 2.1°C which according to figure 7 gives t_c = 22.0°C. A bad balancing would increase the average indoor temperature 22.0 -21.6 = 0.4°C. The influence of bad balancing is higher at low prices of heat.

Figure 4 shows the line of regression equation (4) for indoor temperature in SU-dwellings. t -t^*_m is used to calculate t_m with the help of figure 7 and s = 1.6ºC. t_m is plotted in figure 4 as t_c where it can be compared with the measured values from 1952, 65, 71, 82-85 and 92. The measured values are approximately 0.5ºC higher than the calculated values.

CONCLUSIONS

The calculated averages indoor temperature t_m according to this theory is app. 0.5ºC lower than the measured values. The theory assumes that the owner or the person who sets the temperature controller knows the demand curves for all the households in the building. To get this knowledge he needs to negotiate with all the households. In reality he has contact only with those who wants to increase their indoor temperature and only sometimes with those who wants to lower their indoor temperature. The feedback to the owner about the indoor temperature in the dwellings is normally bad becourse he has to visit the dwellings to measure the temperature.

The theory explains why the collective temperature is higher than the average temperature in dwellings with cost allocation after individual temperature metering. At low prices of heat the collective indoor temperature goes towards the highest demanded temperature and for temperature metering it goes towards the average demanded temperature.

The demand curve for Swedish MU-dwellings was a straight line in Jönsson, 1997 but in this paper we se in figure 4 that it bends to the right at low prices of heat or at high disposable incomes. Both the cost of heat and the cost of cold will be higher than with allocation after indoor temperature except at high prices of heat or at low disposable incomes. Here only the cost of cold will be bigger than with allocation after indoor temperature but a collective temperature do not need any control work from the households.

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