

T11-O-12: Sustainable technologies

INDIVIDUAL HEATING OF MU-DWELLINGS

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SUMMARY

The heat use in individually heated dwellings is reduced if the dwellings are put together in MU-dwelling buildings. It is not the insulation that is changed it is the balance temperature that is increased. The balance temperature is the temperature in a dwelling with closed heaters. The cost for the building and the used heat at a temperature increase in a dwelling is the heat loss out. It is optimal or it gives the highest standard of living if the marginal cost of temperature is for the heat loss out. This is possible if the measured heat to a dwelling is reduced with the variable part in a two part tariff. The rest of the cost of heat is covered with the fixed part. This is important if individual heat cost allocation is used.

INTRODUCTION

The use of individual heating in MU-dwellings makes it interesting to investigate how the heat transfer between the dwellings influences the households hh. The influence is shown in an example where a MU-dwelling building is put together with SU-dwellings so that the number of walls exposed to the outdoor air is reduced. The heat transfer between the dwellings in a building, the heating system and the economic system affects the standard of living for the hh. The dwellings are heated individually and the hh pays after measured heat to the dwelling.

Friedman (1986) presented the economics of heating. The occupants sets their thermostats to the temperature where marginal cost equals marginal value of indoor temperature. The theory about consumption and hh is explained in any textbook in economics for instance Lipsey and Courant (1996). Suitable exercises with indifference curves and budget lines for indoor temperature in Bergstrom and Varian (2006). The influence of indoor temperature and the economic system has been shown with indifference curves and budget lines in Jönsson (2005). Optimal two-part tariffs has been described in Jönsson (2007) and Jönsson (2008). A to high marginal cost gives a to low indoor temperature. The low temperature gives an inconvenience with a value much higher than the value of having individual temperatures.

Two part tariffs with a fixed part (grundkosten) and a variable part (verbrauchskosten) is used in German heat cost allocation. The variable part is between 70 % - 50 % of the heat use according to law. The variable part is determined by the owner within the limits Kreuzberg and Wien (2005).

METHODOLOGIES

This is a theoretical study based on microeconomics. The theory describes how consumers maximize their utility under a limited budget. The indifference curves in figure 1 show combinations of goods that a consumer or a hh is indifferent between. The budget line shows the combinations of goods that is possible to buy for the limited budget. Value is expressed in All Other goods. AO is a mix of consumer goods and is measured in quantity. This makes the analysis independent of money.

The hh has a quantity $i'1$ of AO' during a short period of time at the indoor temperature t^* . The hh needs more of AO' at a lower temperature to be indifferent between the two combinations of AO' and indoor temperature. The function that gives the combinations of indoor temperature and all other AO' goods that have the same value i' is equation 1. i' is the level of utility. k and the quadratic form are known from Jönsson (2004) and Jönsson (2005) as national averages from Swedish single unit dwellings 1952-1992. If $t > t^*$ then $AO' = i'$. $DI' = DI / 8760$ h. DI disposable income for a hh during a year SEK/hh yr (Swedish Kronor, 8 SEK = 1 USD).

$$i' + \frac{DI' \cdot k}{2} (t^* - t)^2 = AO' \quad (1)$$

If a hh have indoor temperature t^* and the utility level $i'1$ it needs a quantity A of AO' goods to be compensated for the fall of indoor temperature to the outdoor temperature t_o . The quantity A is the cost of cold. If the hh is at utility level $i'1$ and gets the quantity A of AO' at indoor temperature t^* then the hh reaches utility level $i'3$. If the indoor temperature then falls to t the utility level falls to $i'1$. To prevent this fall of utility heating is used. Heating in the short run is to get an indoor temperature t at an outdoor temperature t_o . To get the indoor temperature t energy must be used, this reduces the consumption of AO' goods with $CH(t)$ SEK/h so that only X' remain equation 2.

$$DI' - CH(t) = X' \quad (2)$$

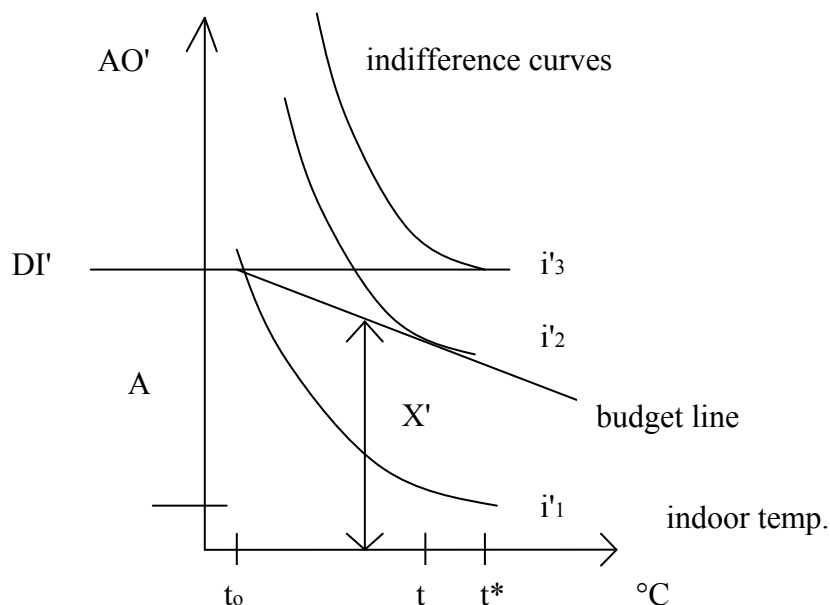


Figure 1 Indifference curves i' in the short run and budget line X' for consumers.

In the following the quantity of AO' goods is replaced by the value of the quantity AO' goods in SEK/h. ph the price of heat SEK/kWh. The cost of heating per time unit CH(t) SEK/h at the indoor temperature t is equation 3.

$$CH(t) = p_h \cdot Q(t) \quad (3)$$

According to physics the heat demand from the heating system in a dwelling at t is Q(t) W equation 4.

$$Q(t) = (\Sigma UA + q \cdot \rho \cdot c_p) \cdot (t - t_o) + \sum_i \Sigma U_j \cdot (t - t_i) - Q_{free} \quad (4)$$

where ΣUA is the sum of heat transmission to the outdoor air $W/^\circ C$, A is the area m^2 , U is the heat transfer coefficient $W/m^2^\circ C$, q is the outdoor air rate for ventilation m^3/s , ρ is the density of air kg/m^3 , c_p is the specific heat of air $kJ/kg^\circ C$, ΣU_j is the sum of specific heat transfer between the dwelling and dwelling i trough the common walls with building parts j $W/^\circ C$, t_i - indoor temperature in surrounding dwelling i $^\circ C$, Q_{free} is the free heat from the sun, light, refrigerator, persons and pipes with no control in the dwelling W.

Equation 4 can be written in a shorter form and if Q_{free} is neglected in equation 5.

$$Q(t) = \Sigma U_o \cdot (t - t_o) + \sum_i \Sigma U_j \cdot (t - t_i) \quad (5)$$

where ΣU_o - the specific heat demand to the outdoor air including transmission and ventilation $W/^\circ C$. The cost of temperature per time unit SEK/h is equation 5 in equation 3.

The marginal cost for temperature per time unit SEK/ $^\circ C$ h is the differential of equation 5 in equation 3 with regard to the indoor temperature equation 6.

$$\frac{dCH(t)}{dt} = p_h \cdot \left(\Sigma U_o + \sum_i \Sigma U_j \right) \quad (6)$$

In the example no U-values of the walls are changed. The walls are only shared with other dwellings. They are moved from facing outdoor conditions to internal walls. Then the sum in equation 6 is constant. Witch makes the cost of a temperature increase constant, independent of how many sides of the dwelling that are facing the outdoor air. The heat loss to the outside is lost heat, the heat exchange with the neighbours is used in the dwelling that gets the heat.

The balance temperature t_b equation 7 is the temperature in a dwelling with closed heaters and heat transfer out and from surrounding dwellings.

$$t_b = \frac{\sum_i \Sigma U_j \cdot (t_i - t_o)}{\Sigma U_o + \sum_i \Sigma U_j} + t_o \quad (7)$$

Individually heated dwellings with individual temperature control figure 2 and heat meter are put together like cubes, so the area exposed to the outdoor temperature is reduced. First a single unit dwelling with 6 sides exposed to the outdoor conditions, then a dwelling in a row

house with 4 sides against outdoor conditions. Finally a MU-dwelling building with only two sides against the outdoor air.

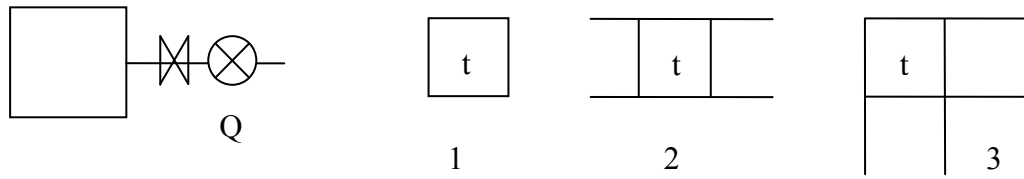


Figure 2. SU-dwelling with individual temperature control and heat meter Q left. Dwelling 1 with 6 sides against outdoor air. Row house 2 with 4 sides against outdoor air. MU-dwelling building 3 with only 2 sides against outdoor air.

The hh in dwelling 1 in figure 3 have no heat demand if the indoor temperature is equal to the outdoor temperature. The heat demand starts in to and the heat cost follows budget line 1. It reaches its highest level 1 at the indoor temperature t_1 .

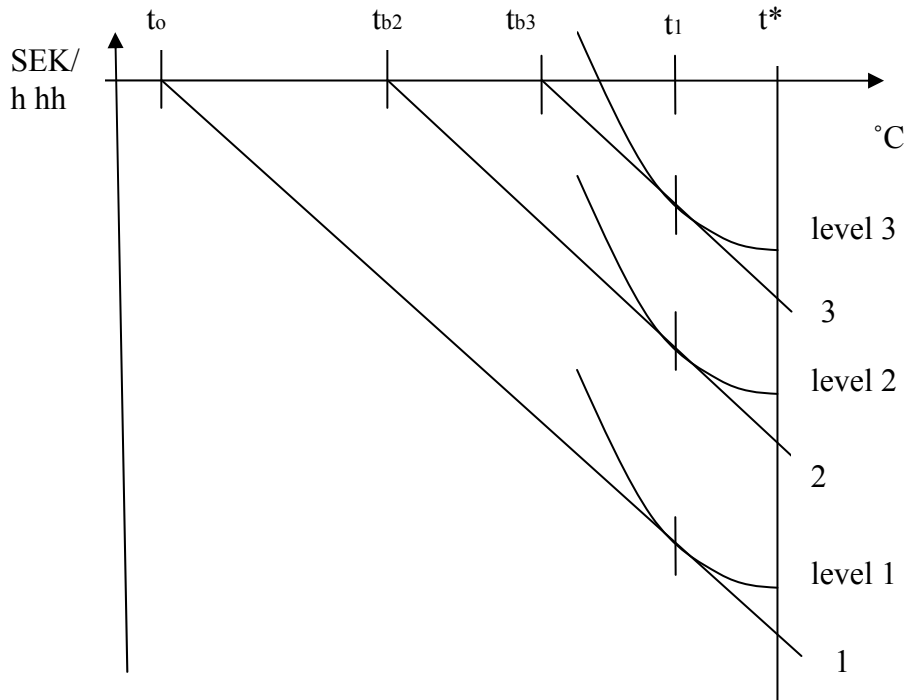


Figure 3. Budget lines and utility levels for the dwelling in the buildings in figure 2

In a row house 2 in figure 2 the hh has no heat demand at the lowest possible indoor temperature the balance temperature tb_2 . It is higher than the outdoor temperature since the dwelling receives heat from surrounding dwellings. Budget line 2 reaches its highest level 2 at the indoor temperature t_1 . The cost of heating is reduced since the number of walls against the outdoor air is reduced. The marginal cost, equation 6 is not changed since the insulation of the walls are not changed. The reduced cost of heating increases the standard of living from level 1 to level 2.

In a MU-dwelling situated in building 3 in figure 2 the hh has no heat demand at tb_3 since the dwellings receives heat from the surrounding dwellings trough 4 sides. The budget line 3

reaches its highest level 3 at the indoor temperature t_1 . The heat loss is reduced but the marginal cost is not changed. The indoor temperature will be the same as before. The reduced cost of heating increases the standard of living to level 3.

To increase the level further it is necessary to reduce the marginal cost of temperature per time unit equation 6. This can be done with a two part tariff. It has a fixed part and a variable part. The marginal cost of temperature per time unit should be for only the heat loss to the outside. The marginal cost of temperature per time unit equation 6 is multiplied with the variable part $(1-\Phi)$ equation 8. This gives equation 9.

$$(1-\Phi) = \frac{\sum U_o}{\sum U_o + \sum_i \Sigma U_j} \quad (8)$$

A dwelling with good insulation against the outside may have $(1-\Phi)$ between 0.2 and 0.25. This means that the marginal cost of temperature according to the heat meter is 4 or 5 times higher than for the heat loss out Jönsson (2007). The rest Φ must be sold as a fixed part. It is difficult to design a fixed part that is independent of the heat use in the building.

$$\frac{dCH(t)}{dt} = p_h \cdot \sum U_o \quad (9)$$

The marginal cost of temperature will be for only the heat loss to the outside, and the budget line 4 in figure 4 points at no heat cost at the outdoor temperature t_o . The lowest possible temperature in the dwelling is still t_{b3} . The low marginal cost makes it optimal for the hh to increase the indoor temperature to t_4 and reach the higher level 4. The building will use more heat at t_4 than at t_1 but the temperature increase is more worth than the cost increase.

If the marginal cost of heat is reduced even more then the hh will chose a higher temperature than t_4 but the cost for the heat loss out follows budget line 4. Line 4 reaches lower levels than level 4. A temperature increase over t_4 is less worth than the cost increase.

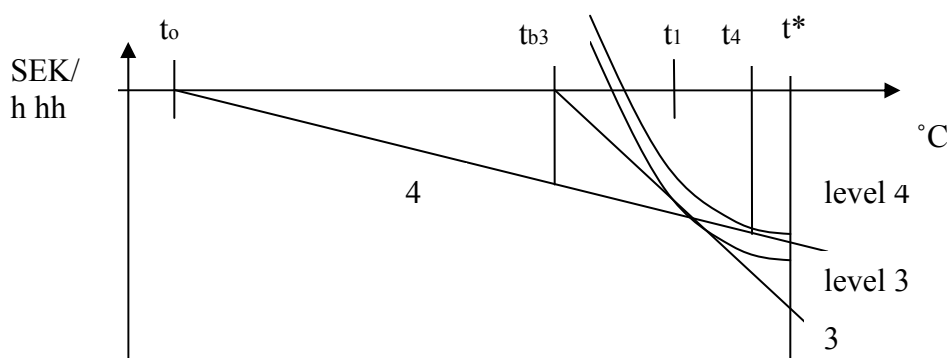


Figure 4. Budget lines and levels with reduced marginal cost of temperature in a two part tariff.

Equation 9 says that the heat to the hh according to measurement should be sold at a lower price than the market price. This is possible if the heat only can be used for heating. Suitable heating systems is a circulation water heating system with heaters in the dwellings.

The Friedman Dwelling

Friedman (1986) introduced a MU-dwelling figure 5 that has no thermal contact with the outdoor air. It only exchanges heat with its neighbours. It was used to demonstrate the marginal cost of temperature. Since the dwelling don't lose any heat out the heating ought to be free.

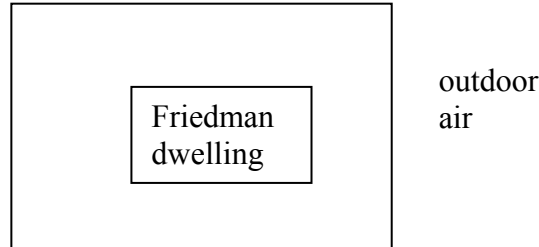


Figure 5. A Friedman dwelling has no thermal contact with the outdoor air. Only heat exchange with other dwellings.

It is a theoretical case since the ventilation air needs heating. The balance temperature in figure 6 t_{bf} depends on the temperatures in the surrounding dwellings. If it is lower than t^* then the hh wants to increase the indoor temperature. The heat used to increase the temperature in the Friedman dwelling is lost to and used in the surrounding dwellings. If the hh pays the full cost of heat and increases the temperature the cost for the hh will follow budget line 5 and the highest level that the hh can reach is level 5 and the optimum temperature is t_5 which is lower than t^* .

If a two part tariff is introduced then the variable part $(1-\Phi)$ equation 8 is zero since there is no thermal contact with the outdoor air $\Sigma U_o = 0$. The heat in the dwelling is free and the hh reaches t^* . The heat transfer at t^* to other dwellings will correspond to 6 at the budget line 5 but the heat is free so the hh will reach level 6. The fixed part is the same as for the other dwellings. A fixed part will reduce the level but it will not reduce the indoor temperature from t^* .

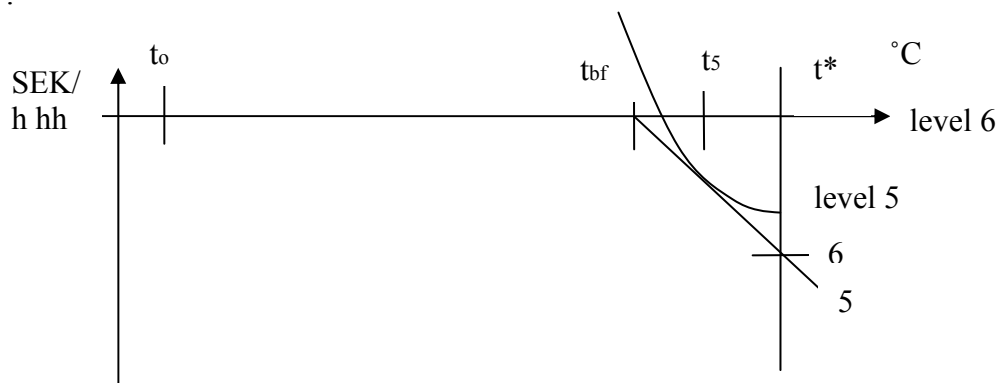


Figure 6. Budget lines and utility levels in a Friedman dwelling. It has no thermal contact with the outdoor air.

RESULTS AND DISCUSSION

The cost of heat for indoor temperature is reduced if the dwellings are put together in a MU-dwelling building but the marginal cost of temperature is not changed. The reduction of the area against the outdoor air increases the balance temperature of a MU-dwelling. The balance temperature is the temperature in a MU-dwelling with closed heaters, heat transfer from surrounding dwellings and heat loss out.

The cost of heat from the heating system is determined by the marginal cost of temperature, the balance temperature and the indoor temperature. The marginal cost and the balance temperature depends on the heat transfer data of the building and on the temperatures in the other dwellings. It is only the indoor temperature that the household determine. Since a hh depends on the other hh:s it is natural to cooperate about the heating and to reduce the marginal cost of temperature for a hh to what corresponds to the heat loss to the outdoor air. The marginal cost of temperature may be 4 or 5 times higher than for the heat loss out. A cooperation to change the economic system for heat cost allocation will increase the standard of living for the hh.

If the household pay a fixed part for heating that is used to reduce the price of heat then the marginal cost of temperature is reduced and the hh will chose a higher indoor temperature that will give a higher standard of living.

CONCLUSIONS

To make it possible for a household to reach the highest standard of living with available resources the heating must be designed so both the technical and the economic system fits human demand.

The MU-dwellings protects each other from heat loss to the outdoor air but not from heat exchange with each other. The heat loss out determines the heat use for the whole building. The heat exchange determines the marginal cost of temperature for a MU-dwelling. The consequence of the heat exchange can be reduced if the value of the exchanged heat is reduced in a two part tariff, a fixed and a variable part.

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