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HEAT COST ALLOCATION IN TWO BUILDINGS WITH COLLECTIVE TEMPERATURES

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SUMMARY

This is a theoretical study based on the theory of collective goods from microeconomics. If a MU-dwelling building have supply water temperature control SWTC and heat metering HM the collective indoor temperature will be 21.7°C. If two identical buildings share HM and both have their own SWTC then the indoor temperature in both buildings will be 22.5°C since the marginal cost of temperature for the collectives is halved. If the two buildings have less than 15 dwellings each one HM and one SWTC can be used for both buildings. The difference in temperature between the buildings will probably be less than 1°C. If they are bigger it is profitable to invest in one HM and one SWTC for each building to avoid the low indoor temperature from the unbalance in heat demand.

INTRODUCTION

The application of the theory of collective goods from economics to indoor temperature makes it possible to analyse indoor temperatures that are common to many households hh and to find the best combination of supply water temperature control SWTC and heat metering HM.

A collective indoor temperature is common to two or more hh. A collective indoor temperature is used if a building with MU-dwellings have a common heating system where the supply water temperature is automatically controlled after the outdoor temperature. The heaters gives the same temperature in all dwellings in the collective. The heat use is measured for the collective and the cost of heat is distributed after dwelling area to the hh.

A theory for the choice of the best quantity of collective or public goods was developed by Erik Lindahl 1919 according to Bergstrom (2005). Ståhl (1975) called heating a semi-collective goods. Semi means that the access to the indoor temperature is limited to the members of the hh. He assumed that the control of the indoor temperature is individual but it is in practice collective. The theory for collective goods was explained in Bohm (1977). Friedman (1986) used the theory for public goods to find the best common indoor temperature in a building with two dwellings.

The demand curve for indoor temperature in Swedish multiple unit dwellings was determined by Jönsson (1997) as a straight line and in Swedish single unit dwellings by Jönsson (2004). The demand curve for a collective indoor temperature was derived by Jönsson (2006). The theory of collective indoor temperature can be used to calculate the profitability with individual heat cost allocation.

METHODOLOGIES

This is a Theoretical Paper Based on Methods From Microeconomics.

A household in a MU-dwelling produces indoor temperature with heat and insulation. The hh balances the cost of heat or cost of temperature horizontally marked in figure 1 and the cost of cold diagonally marked against each other. The hh choses the indoor temperature where the marginal cost of heat, supply is equal to the marginal cost of cold, demand. The marginal cost of heat is the cost of a temperature increase.

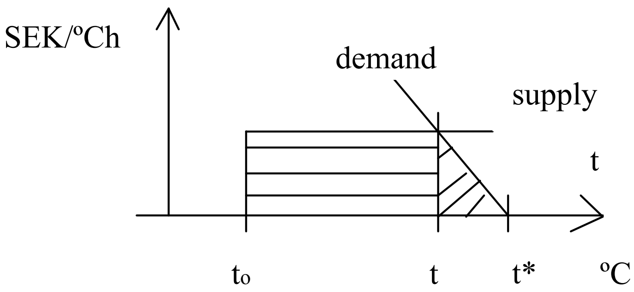


Figure 1. Demand and supply curves for indoor temperature. Indoor temperature t. Outdoor temperature to. Highest indoor temperature the household is willing to pay for t*.

Figure 2 shows the sum of demand curves for the hh:s in a building. The area under the demand curve can be seen as a cost of cold. The sum is the sum of cost of cold. The fourth hh will not get any cost of cold at tc. The optimal collective temperature tc minimises the sum of sum of cost of cold and sum of cost of heat for all hh. If there are 4 hh the sum of cost of heat is 4 times the area ph ΣUo (tc –to). ph price of heat SEK/kWh. ΣUo specific heat demand for heat loss out W/˚C. (Swedish Kronor, 8 SEK = 1 USD)

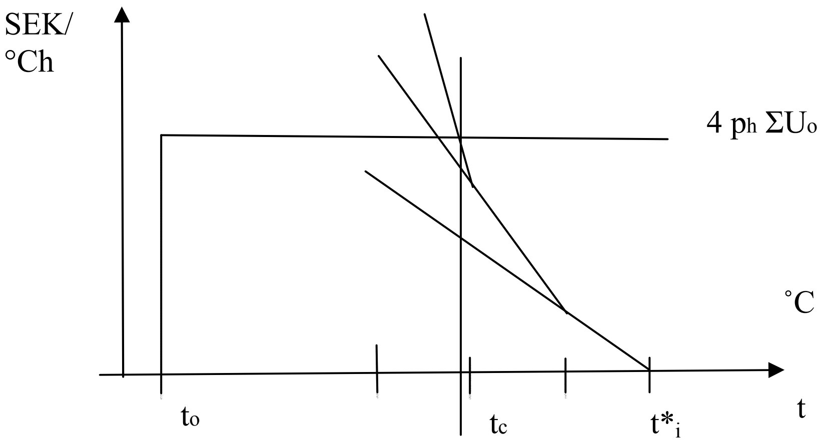


Figure 2. The sum of demand curves. Collective temperature t_c where the sum of demand curves reaches the sum of marginal costs for four households

The dimensionless demand curve for a collective indoor temperature if t*i belongs to a normal distribution is found from a numerical integration of the normal distribution. The area under the demand curve or the dimensionless cost of cold is found after another integration. To calculate the cost of cold for a collective temperature the dimensionless demand curve in Table 1 can be used. It is an improved version of Jönsson (2007). The t* for the hh are normally distributed with the standard deviation s* = 1.3°C Holgersson and Norlen (1983). t^*_m = 21.65°C and k = 0.004 1/°C^2 from the indoor temperature in Swedish SU-dwellings 1952 - 1992 from Jönsson (2004) and Jönsson (2005).

The dimensionless indoor temperature from equation (1).

$$t^\circ = \frac{t^*_m - t}{s}$$
(1)

The dimensionless reduction of indoor temperature equation (2).

$$t^\circ_i = \frac{p_h \cdot \Sigma U_o}{k \cdot DI' \cdot s}$$
(2)

The standard deviation s from both the variation in preferences s* and from the variation in temperature between the dwellings s_b from equation (3).

$$s = \sqrt{s^{*2} + s_b{}^2}$$
(3)

The marginal cost of cold MC SEK/°Ch, hh from equation (4).

$$MC = MC^\circ(t^\circ_c) \cdot k \cdot DI' \cdot s$$
(4)

The costs of cold is CC SEK/h, hh from equation (5). To get the cost per year multiply with the length of the heating season τ h/year.

$$CC = CC^\circ(t^\circ_c) \cdot k \cdot DI' \cdot s^2$$
(5)

This theory can be used to study how changes in marginal cost influence the collective temperature in MU-dwelling buildings. The marginal cost is changed with different arrangements of SWTC and HM.

Two identical buildings are used with different combinations of temperature control and heat metering. They are built after 1980 in Sweden. The U-values and the ventilation rates were reduced after the energy crisis 1973. Heat loss from a MU-dwelling to outdoor air (transmission and ventilation) ΣU_o = 44 + 30 = 74 W/°C. The dwelling area is 70 m^2/dw. The economic data for the household refers to 1992. The average disposable income for a household in a MU-dwelling were 135 000 SEK/yr hh. The disposable income per hour is DI' = DI/8760 = 15 SEK/h, hh. The price of heat ph = 0.5 SEK/kWh (including tax). The hh:s have average preferences for indoor temperature. The standard deviation for indoor temperature for the dwellings in a building is sb = 0.8 °C Holgersson and Norlen (1983) witch gives s = $(1.3^2 + 0.8^2)^{0.5}$ = 1.6 °C equation 3.

Table 1. Dimensionless average of individual t°i and the collective temperature t°c marginal cost of cold MC°(t°c) and cost of cold,CC°(t°c).

$t°_i$	$t°_c$	$t°_c$	MC°(t°c)	CC°(t°c)
3	3	3	3	5
2.5	2.5	2.5	2.5	3.62
2.0	1.99	2.0	2.01	2.49
1.8	1.78	1.8	1.81	2.11
1.6	1.58	1.6	1.62	1.77
1.4	1.36	1.4	1.44	1.46
1.2	1.14	1.2	1.26	1.19
1.0	0.90	1.0	1.08	0.96
0.8	0.65	0.8	0.92	0.76
0.6	0.35	0.6	0.77	0.59
0.4	0.0	0.4	0.63	0.45
0.3	-0.21	0.2	0.51	0.34
0.2	-0.5	0.0	0.40	0.25
0.1	-0.9	-0.2	0.31	0.17
0.05	-1.25	-0.4	0.23	0.12
0.025	-1.56	-0.6	0.17	0.086
0	-3	-0.8	0.12	0.057
		-1.2	0.056	0.023
		-1.6	0.023	0.008
		-2	0.008	0.002
		-3	0	0

Two BuildingsWith SWTC and HM in Both Buildings

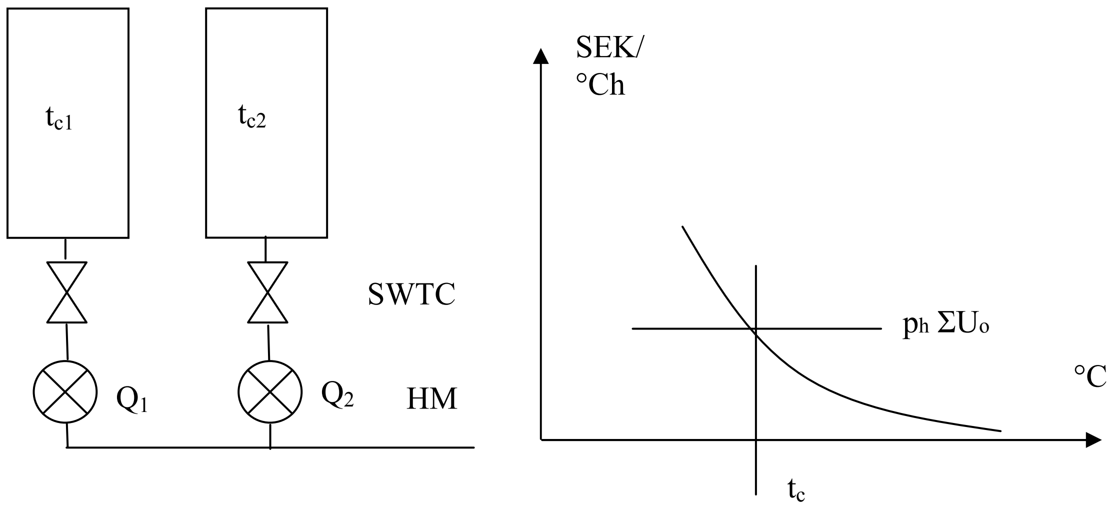
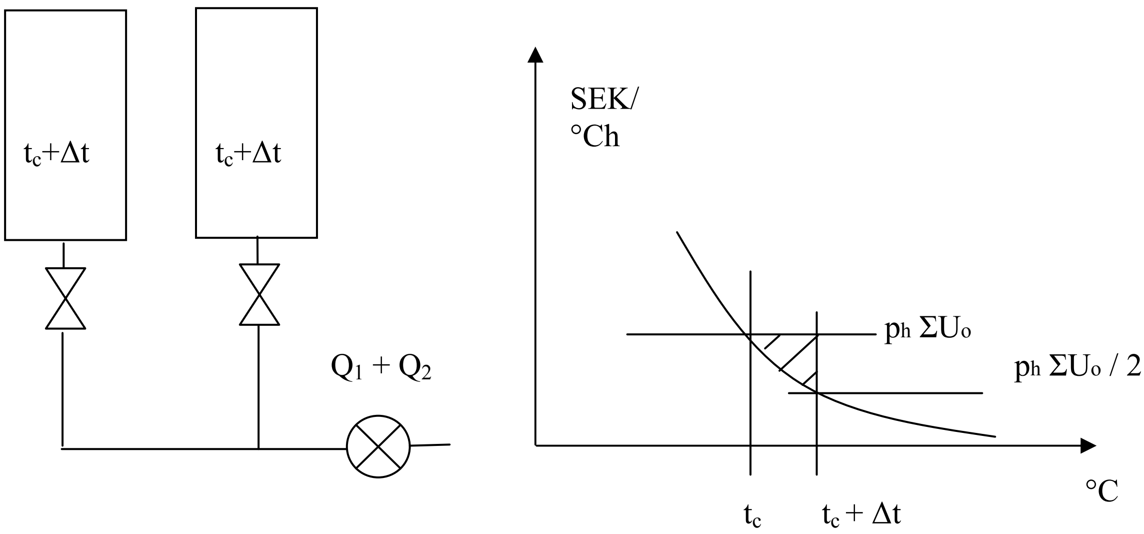


Figure 3. Two identical buildings with collective temperatures tc1 and tc2 with supply water temperature control SWTC and heat meter HM in both buildings. The demand curve for a collective indoor temperature.

Equation 2 gives $t°_i = 0.38$ and Table 1 gives $t°_c = - 0.05$. Equation 1 give $t_c = 21.7°C$. According to the normal distribution N(-0.05, 0, 1) = 48 % of the households feel a cost of cold. This is the percentage disturbed by a low indoor temperature. Both buildings will get the collective indoor temperature they demand. If both the buildings and the households are identical then tc1 = tc2 will be identical 21.7˚C. There is no loss in relation to the ideal collective temperature.

Two Buildings With SWTC in Both Buildings and Shared HM

If two buildings share the same heat meter the marginal cost of temperature is halved. The collectives share the cost of a temperature increase independent of in what building the increase is done.



Figur 4. Two identical buildings with collective temperatures with SWTC in both buildings and shared HM. Demand curve for a collective indoor temperature. Δt temperature over the ideal collective temperature tc.

The marginal cost ph ΣUo/2 in equation 2 gives $t°_i$ = = 0.19 and Table 1 gives $t°_c = - 0.54$ Equation 1 gives $t_c = 22.5°C$. Shared HM increases the indoor temperature from 21.7°C to 22.5°C. According to the normal distribution N(-0.54, 0, 1) = 30 % of the households have a cost of cold. At 21.7°C 48 % of the households had a cost of cold.

The loss from extra cost of heat in relation to the optimal tc is the marked triangle in figure 4.
To calculate the area of the triangle MC° from Table 1 is linearised round t°c = -0.05 to
$MC° = 0.38 – 0.4 * \Delta t°$
The dimensionless temperature difference, equation 1: $\Delta t° = \Delta t/1.6°C$. 0.8/1.6 = 0.5
The dimensionless area of the triangle CC° is $0.40 * \Delta t°^2 / 2 = 0.40 * 0.5^2 / 2 = 0.05$
The loss from the extra cost of heat equation 5 is 46 SEK/yr hh.

If the buildings are identical then the indoor temperature will be 0.8°C higher than with two HM. If tree buildings share one HM then the marginal cost will be reduced to a third and the indoor temperature $t°_c = - 0.78$ will be 22.9°C.

Shared SWTC and HM

In this combination it is the same SWTC for both buildings but it is not possible to get exactly the same temperature in both buildings. The difference in average temperature from t_c is Δt. It may come from differences in the outdoor air rates and from differences in the U-values of the walls between the buildings. A heat balance calculation at 0°C outdoor temperature gives that a 10 % higher outdoor air rate than the design outdoor air rate will give 0.5°C lower indoor temperature. This is a possible unbalance in outdoor air rate. Instead of 21.7°C the temperature would be 21.2°C.

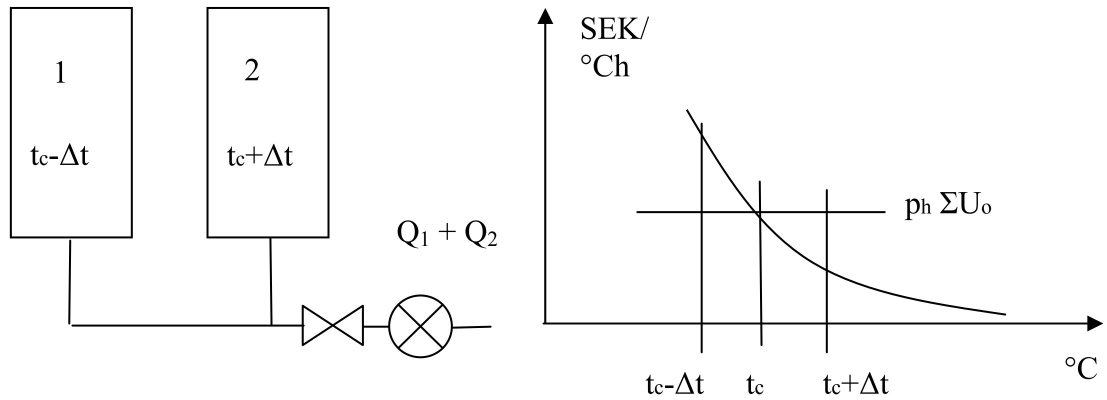


Figure 5. Two identical buildings with shared SWTC and HM and demand curve for a collective indoor temperature. Δt unbalance in indoor temperature from tc.

Building 1 left will get the indoor temperature tc - Δt but pays for tc. The extra inconvenience is the diagonally marked area in figure 6. Building 2 will get the indoor temperature tc + Δt but pays for tc. The extra benefit is the area under the curve up to tc + Δt in figure 6. The hh in building 1 will get less than they pay for and in building 2 they will get more than they pay for.

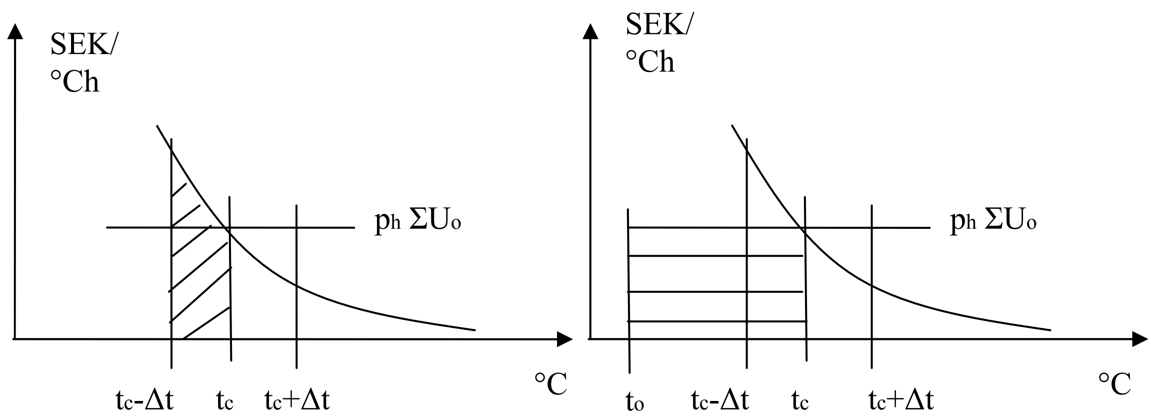


Figure 6. Inconvenience of not delivered indoor temperature in building 1 already paid for, diagonally marked. Paid indoor temperature, horizontally marked.

The hh:s in building 1 may as well pay the value of the extra inconvenience for the investment in an extra HM and a SWTC. Then they can get the temperature tc.

The dimensionless marginal cost MC° is linearised round t° = -0.05 as: 0.38 + 0.40 Δt°
The extra dimensionless area for building 1: Δt = 0.5°C Δt° = 0.5 / 1.6 = 0.31
(0.38 + 0.40 *0.31 /2) * 0.31 = 0.14

Equation 5 gives the value of the low indoor temperature to 130 SEK/yr hh

A HM and SWTC costs about 15 000 SEK in investment or 2 000 SEK/yr including operation. If building 1 has n hh:s they can pay n x 130 SEK/yr hh = 2 000 SEK/yr to get a SWTC and HM instead of a low indoor temperature. This gives n = 15 hh. If there are more than 15 dwellings in each building it is better to have HM and SWTC in both buildings.

RESULTS AND DISCUSSION

If both buildings in Table 2 have SWTC and HM the collective indoor temperature will be 21.7°C. If the buildings share both SWTC and HM then the unbalance in heat demand will give the buildings different temperatures. The extra loss is the loss from a low temperature at 21.2°C in the cold building over the loss at 21.7°C. If two buildings share the same HM the marginal cost of temperature is halved and the indoor temperature in both buildings is increased to 22.5°C.

Table 2. Collective indoor temperature tc in building 1 and 2 with combinations of supply water temperature control SWTC and heat meter HM. Extra loss in relation to SWTC and HM in both buildings.

Control	Meter	t_{c1}	t_{c2}	Extra loss	
SWTC	HM	°C	°C	SEK/yr hh	
2	2	21.7	21.7	0	
1	1	21.2	22.2	130	in build. 1
2	1	22.5	22.5	46	

Two buildings with MU-dwellings should have HM and SWTC in each building if they have more than 15 dwellings. The heat cost should be distributed after area in each building. If the two buildings have less than 15 dwellings each one HM and one SWTC can be used. The difference in temperature between the buildings will probably be less than 1°C. The extra loss in Table 2 is over the loss at the ideal collective temperature.

Shared HM and a SWTC in each building should be avoided.

CONCLUSIONS

Different combinations of supply water temperature control SWTC and heat meter HM in two MU-dwelling buildings will give different indoor temperatures depending on changes in the marginal cost and depending on unbalances. The indoor temperature as a result of the change in marginal cost of temperature can be calculated with the theory for collective goods.

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