THE LOSS OF WORK HOURS IN AN ODOR ACCORDING TO WEBER-FECHNERS LAW

A. Jönsson
Secondary Vocational Training, Härnösand, Sweden

Abstract

This paper derives a theory for the loss of work hours as function of the concentration of a substance that gives an odor. Weber and Fechner described the human perception, psycho of physically measurable properties during the 19-th century (psychophysics). The perception is logarithmic to the property and the odor threshold describes at which concentration an individual feels an odor and how an individual feels an increase in concentration.

The value of the logarithm in the Weber-Fechner law is the odor or the individual demand for reduced pollution. The individual demand and the distribution of odor thresholds give the average demand for reduced pollution in a collective. The loss of production is the sum of demands for improvement. The demands are added in an integration of the W-F law, which gives the function for loss of production. It is a curve. The dissatisfied at different outdoor air rates and a distribution of odor thresholds is used to determine the threshold for the most sensible individual.

The theory is calibrated against the marginal cost of concentration for the well-known odor body odour at an economically optimal outdoor air rate. The calibrated function is used to calculate the loss of production for a new pollutant where the loss of work hours has been measured in a work rate study. The derived functions shows both the optimal loss in a collective from body odor at 10-15 l/s pers and the measured loss in body and carpet odor at 3 and 10 l/s pers.

Keywords: outdoor air rate, optimum, economics, Weber-Fechner, loss

1 Introduction

Fechner (1860) used the logarithm to determine the sensation of physical properties like noise and light. Yaglo, Riely & Coggins, 1936 found that the average vote from a trained panel followed the logarithm of the outdoor air rate according to Berglund et al (1979). A trained panel votes like one individual. The logarithm of the dose or of the concentration is used in toxicology for the dose-response relation, Lidman (2008). The loss of production as function of the concentration of indoor pollutants in an economic theory was a line in Jönsson (1995). The loss of production in a collective was a curve in Jönsson (2011) since the individuals had a linear loss but the distribution of individual odor thresholds followed a curve.

2 Methods

2.1 Demand of improvement for an individual

A concentration above the threshold c* gives an odor and a loss of production. The odor can be seen as the demand of reduced concentration D(x). If the concentration x is reduced one unit then the loss of production is reduced with D(x). This means that the loss can be calculated if D(x) is integrated over x. An individual is studied first and then many individuals with different odor thresholds in a collective

$$D(x) = a \cdot \ln \frac{c}{c^*} = a \cdot \ln x \qquad (1)$$

The demand of improvement D(x) h/yr pers, 1 of one concentration unit x at c from Eq.(1). It is known as the Weber-Fechner law. a gives D(x) in the unit hours per pers, year and for the improvement of one x. It is shown in Fig. 1 with a = 1, ind. x has no dimension. Since the odor threshold c* is individual then x is individual.

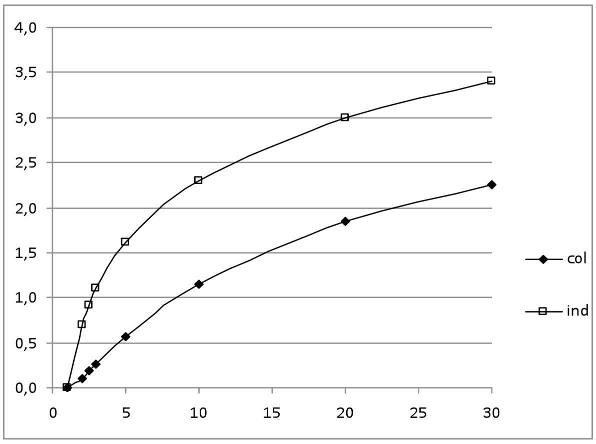

Figure 1. Individual demand of improvement as function of x_1 *for the most sensible individual,* ind. *Average demand of improvement for the members of a collective,* col. *as function of* x_1 *for the most sensible individual*

2.2 Loss of production for an individual

The Lossh(x) h /pers, yr is the sum of D(x) over all x. The demand for every x are added and the sum of demand of improvement is the improvement that will come from a reduction of x down to 1 Eq.(2). The loss of production for the most sensible individual is shown in Fig. 2, ind. a = 1.

$$Lossh(x) = \int D(x)dx = a \cdot \int_1^x \ln x \cdot dx = a \cdot \left(x \cdot \ln x - (x-1)\right) \qquad (2)$$

2.3 Loss of production for a collective

Many individuals are in a room with the concentration c . Every individual has an odor threshold c^*_i. The most sensible has c^*_1 and the least sensible individual has c^*_n. ln x_1 = ln (c/c^*_1) for the most sensible person is used as x-axis for all others odor thresholds Fig. 3. The individuals are numbered after their thresholds.

There are two cases, one when c is below the highest individual odor threshold $x_1 < x^*_n$ and when the concentration is above all individual thresholds $x_1 > x^*_n$, Fig. 3.

Here there is a difference between dissatisfied at first entrance and the odor threshold. If someone decides that the odor is acceptable he has first registered the odor and then evaluated if it is acceptable or not.

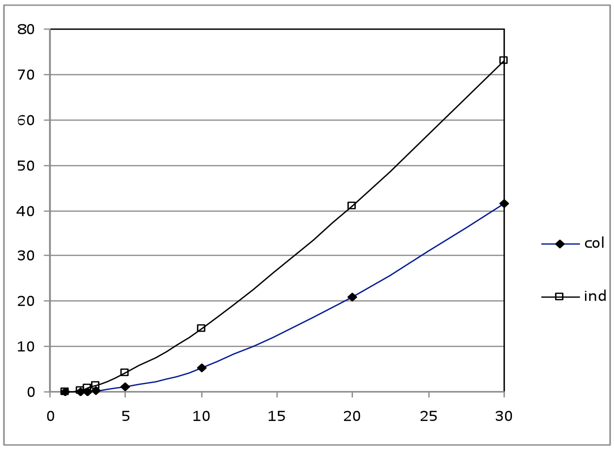

Figure 2. Loss of production for an individual Lossh(x) ind. *and Loss of production in average for a collective,* col. *as function of* x_1 *for the most sensible individual*

It is a simplification to use a line for the odor threshold distribution in the logarithmic diagram, Fig. 3. It is a normal distribution according to Lidman (2008).

The number of dissatisfied, dis at x_i follows Eq.(3). The odor threshold c^*_i in relation to c^*_1 for the dis person is Eq.(4).

$$dis = \frac{\ln x_i}{\ln x_d} \qquad (3)$$

$$\frac{c^*_i}{c^*_1} = x^*_i = x^*_n{}^{dis} = \left(\frac{c^*_n}{c^*_1}\right)^{dis} \qquad (4)$$

Distribution of dissatisfied according to Eq.(5).

$$x_i = x_d{}^{dis} = \left(\frac{c_d}{c^*_1}\right)^{dis} \qquad (5)$$

If the most insensible is dissatisfied at c_d = 100 * c^*_1 and if the most insensible has an odor threshold at c^*_n = 10 * c^*_1 then the relation between odor threshold and concentration for dissatisfied follows Eq.(6).

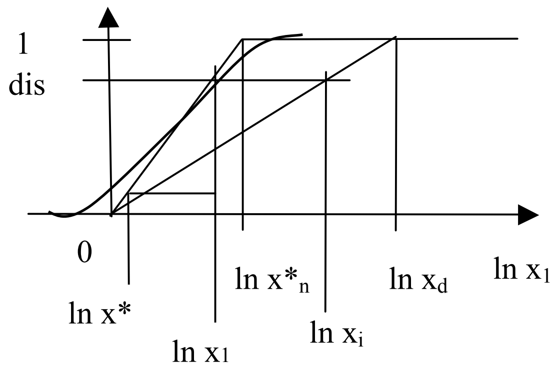

Figure 3. Distribution of odor thresholds x and dissatisfied at first entrance* x_d *as function of* ln x_1. *The normal distribution of odor thresholds is approximated with a line*

$$x^*_n = x_d{}^{0,5} \qquad (6)$$

For example x_i for the 20 % dissatisfied is 100^0,20 = 2,5 Eq.(5) and if the odor threshold for the most sensible is c^*_1 = 150 ppm then 20 % is dissatisfied at 2,5 * 150 = 375 ppm above outdoors. The odor threshold c^*_i for the 20% individual is $x^* = 2{,}5^{0,5}$ = 1,58 Eq. (6) and c^*_i = 150 *1,58 = 237 ppm above outdoors.

2.4 Average demand of improvement in a collective, $x_1 < x^*_n$

The demand for the individual, dis is proportional to the distance (ln x_1 – ln x*) according to Eq. (1). The average demand of improvement, D(x_1) h/pers yr 1 is the area of the triangle 0, ln x_1, dis times a in Fig. 3. and Eq.(7).

$$D(x_1) = a \cdot \frac{dis \cdot \ln x_1}{2} = a \cdot \frac{\ln x_1 \cdot \ln x_1}{2 \cdot \ln x^*_n} \qquad (7)$$

The average demand of improvement in a collective in Fig. 1, col, a = 1 x_1<10 is lower than the demand for the most sensible individual, since only a part of the collective feels the odor. The factor a is the same for all individuals.

2.5 Average loss of production for all individuals in a collective, $x_1 < x^*_n$

The sum of demand for improvement or the loss of production in average per person as function of x_1 for the most sensible individual from x = 1 to x_1, is the integral of Eq.(7) in Eq.(8) and Fig. 2 x_1<10, a = 1. CRC Handbook (1980)

$$Lossh(x_1) = \frac{a}{2 \cdot \ln x^*_n} \cdot \int_1^{x1} (\ln x_1)^2 dx_1 = \frac{a}{2 \cdot \ln x^*_n} \cdot \left(x_1 \cdot \ln x_1 \cdot (\ln x_1 - 2) + 2 \cdot (x_1 - 1)\right) \qquad (8)$$

2.6 Average demand of improvement in a collective, $x^*_n < x_1$

Above x^*_n or above the concentration c^*_n all individuals feel the odor and an increase of c gives a higher number of dissatisfied. The average demand of improvement is Eq.(9) in Fig. 1 10<x_1.

$$D(x_1) = a \cdot \left(\ln(x_1) - \frac{\ln(x^*{}_n)}{2} \right) \quad (9)$$

2.7 Average loss of production for all individuals in a collective, $x^*_n < x_1$

Above x^*_n or above the concentration c^*_n all individuals feel the odor. The average loss of production is Eq.(10) in Fig. 2, $10<x_1$.

$$Lossh(x_1) = a \cdot \left(x_1 \cdot \ln x_1 - x_1 + \frac{(x^*{}_n - 1)}{\ln x^*{}_n} - \frac{x_1 \cdot \ln x^*{}_n}{2} \right) \quad (10)$$

2.8 Determination of a in body odor

a is chosen so that the optimum outdoor air rate for body odor will be 10, 12 resp. 15 l/s pers. The investment in a ventilation system with 1,25 m^3/s supply and return air (100% outdoor air) with heat recovery in an office building with 50 rooms is 1 MSEK Jönsson (1995). If half the investment depends on the outdoor air rate and the annuity is 0,074 then the cost of capital is 30 000 SEK/m^3/s, yr. The cost of operation (heat and maintenance) is 7 400 SEK/ m^3/s, yr during 2000 h/yr work hours. Then it will cost l = 37,4 SEK/ l/s yr to increase the outdoor air rate at the design of the building. The fixed investment is 500 000 / 50 room * 0,074 = 740 SEK/ room yr. The source strength of CO_2 is f = 16 l/h, pers = 0,0044 l/s pers Eq.(11).

$$K(q) = m + l \cdot q = m + \frac{l \cdot f}{c} = 740 + \frac{0{,}17}{c} \quad (11)$$

The price of an input hour is 250 SEK/h and the odor threshold for the most sensible is c^*_1 = 150 ppm = 0,00015. The cost in h/yr pers, $c = x_1 * c^*_1$ in Eq.(12).

$$K(x_1) = 3 + \frac{4{,}53}{x_1} \quad (12) \qquad \frac{dK(x_1)}{dx_1} = -\frac{4{,}53}{x_1^2} \quad (13)$$

The average demand of improvement in a collective $D(x_1)$ from Eq.(7) is compared with the derivative of the cost in hours per pers and year in Eq.(13) This gives a for the assumed optimum outdoor air rates in Table 1. The curves for $Lossh(x_1)$ in Fig. 5.

Table 1: *Individual concentration unit x_1 for body odor, derivative of cost in hour/ yr pers and average demand of improvement for a collective $D(x_1)$*

optimal air rate	x_1	$dK(x_1)/dx_1$	$D(x_1)$	a
l/s				h/yr, pers
10	3	0,50	0,26	1,92
12	2,5	0,72	0,18	4
15	2	1,13	0,10	11,3

2.9 Average loss of production in body+carpet odor

The new body+carpet odor gives a higher share of disturbed dis_2 than body odor and it is assumed to have the same relation between c^*_1 and c_d as body odor or $c_d/c^*_1 = 100$. This makes the lines for the distribution of dissatisfied for both odors parallel in Fig. 4. Measured percentage for dissatisfied in Table 2 and approximated lines in Eq.(14) and Eq.(15). At 30 and 74 l/s pers no one can feel the body resp. body and carpet odor.

b. odor $$dis = \frac{\ln 30 - \ln q}{\ln 100} \quad (14)$$

b.+c. odor $\quad dis_2 = \dfrac{\ln 74 - \ln q}{\ln 100}$ (15)

Body odor has the threshold for the most sensitive at c^*_1 = 150 ppm CO_2 above outdoors and b.+c. odor has the threshold c^*_1 = 60 ppm CO_2 (150 * 30/74 = 60 ppm). x_1 at 3 l/s pers for b.+c. odor then becomes x_1 = 24,7 (74/3 = 24,7).

Wargocki et al. (2000) determined the productivity in b+c odor at 3 outdoor air rates, Table 2. The measured loss in Fig. 5 is the percent loss from Table 2 of 2000 h/year. The calculated functions and measured loss in Fig. 5. Long term effects on health are not included.

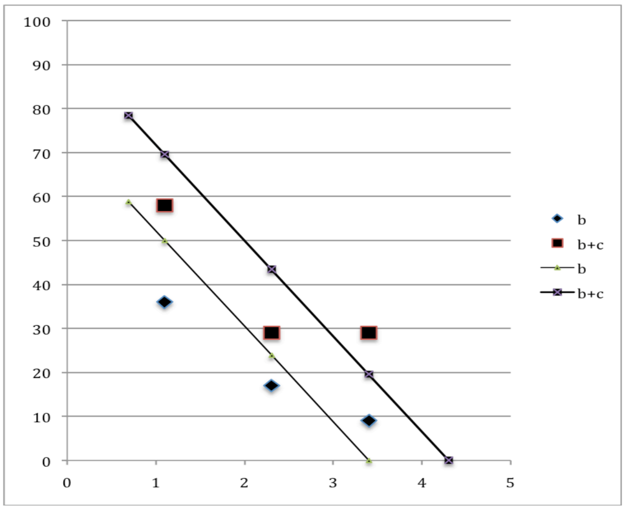

Figure 4. Distribution of dissatiesfied for the individuals in a collective as function of ln(q/1) *for b. and b.+c. odor*

Table 2: *Percentage dissatisfied, production and loss at different outdoor air rates*

Outdoor	Dissatisfied		Productivity		
air rate	body	body+carpet	Speed, norm.	body+carpet	
	Knudsen	Wargocki	Typing	Add	Pro read
l/s pers	%	%	: %	: %	: %
3	36	58	141,8: 3,6	221,1: 7,5	5,05: 7,1
10	17	29	144,6: 1,6	232,9: 2,1	5,28: 2,4
30	9	29	146,9	237,7	5,41

3 Results

The logarithm in Weber-Fechners law represents the demand of reduced concentration. The loss of production is the sum of demand of reduced concentration. The demand is added in an integral of the logarithm. The integral is tangential to the function for the cost of concentration at the economic optimum. This condition gives the loss of production in work hours per year as function of the concentration and of the odor threshold.

The function that gives the optimal loss in body odor at 10 l/s, pers x_1=3 also gives the measured loss for typing at 3 l/s pers in body+carpet odor x_1=24,7. The function that gives the optimal loss in b. odor at 12 l/s, pers x_1=2,5 also gives the measured loss for addition and proof reading at 3 l/s pers in b.+c. odor x_1=24,7. The function that gives the optimal loss in b. odor at 15 l/s, pers x_1=2 also gives the measured loss for typing, addition and proof reading at 10 l/s pers in b.+c. odor x_1=7,4. If it is economically optimal to use 10 - 15 l/s pers in body odor then the measured losses of production in the work rate study are realistic.

4 Conclusion

The function that is derived with an integration of the logarithm in Weber-Fechners law and with the assumption that the distribution of odor thresholds in a collective is linear gives the optimal loss in body odor at the used outdoor air rates 10 - 15 l/s pers. It gives the measured loss in a mixture of body and carpet odor at 3 and 10 l/s pers. Next step is to compare with the results from other work rate studies, with measurements of odor and building odor, with time series of used or recommended outdoor air rates and to use a normal distribution for dissatisfied in the logarithmic diagram.

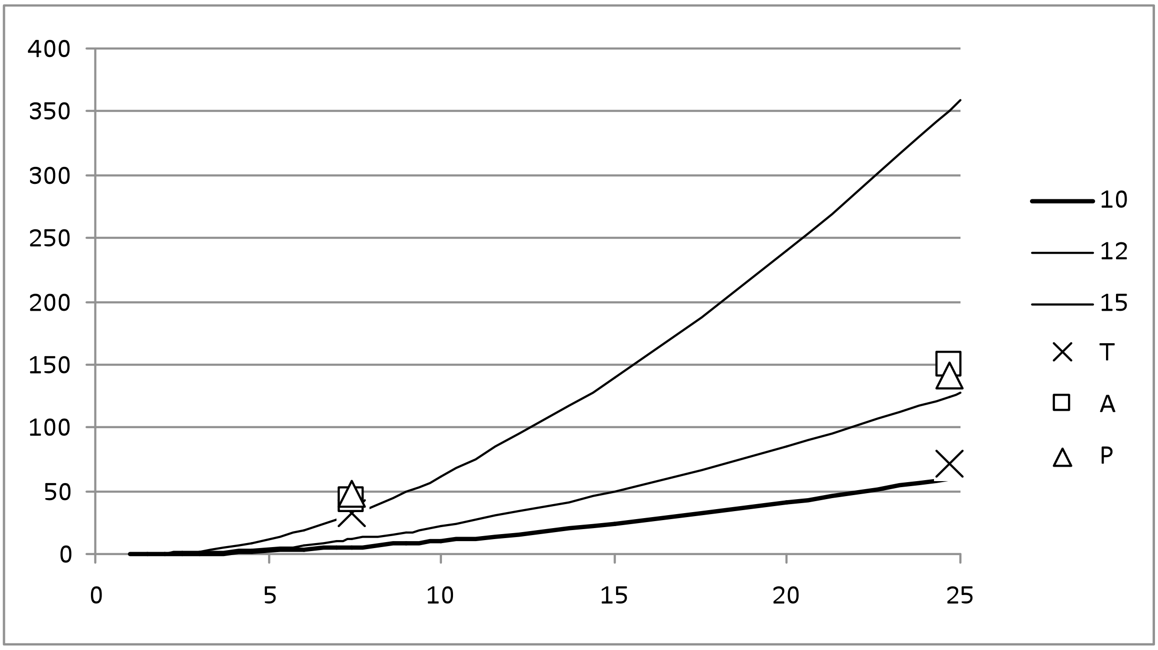

Figure 5. Loss of production in h / yr pers as function of $x_1 = c/c^*_1$. *Curves for loss as function of* x_1 *from theory for the optimal out door air rates 10, 12 and 15 l/s pers. Points according to the work rate study, Table 2 for Typing, Addition and Proof reading*

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